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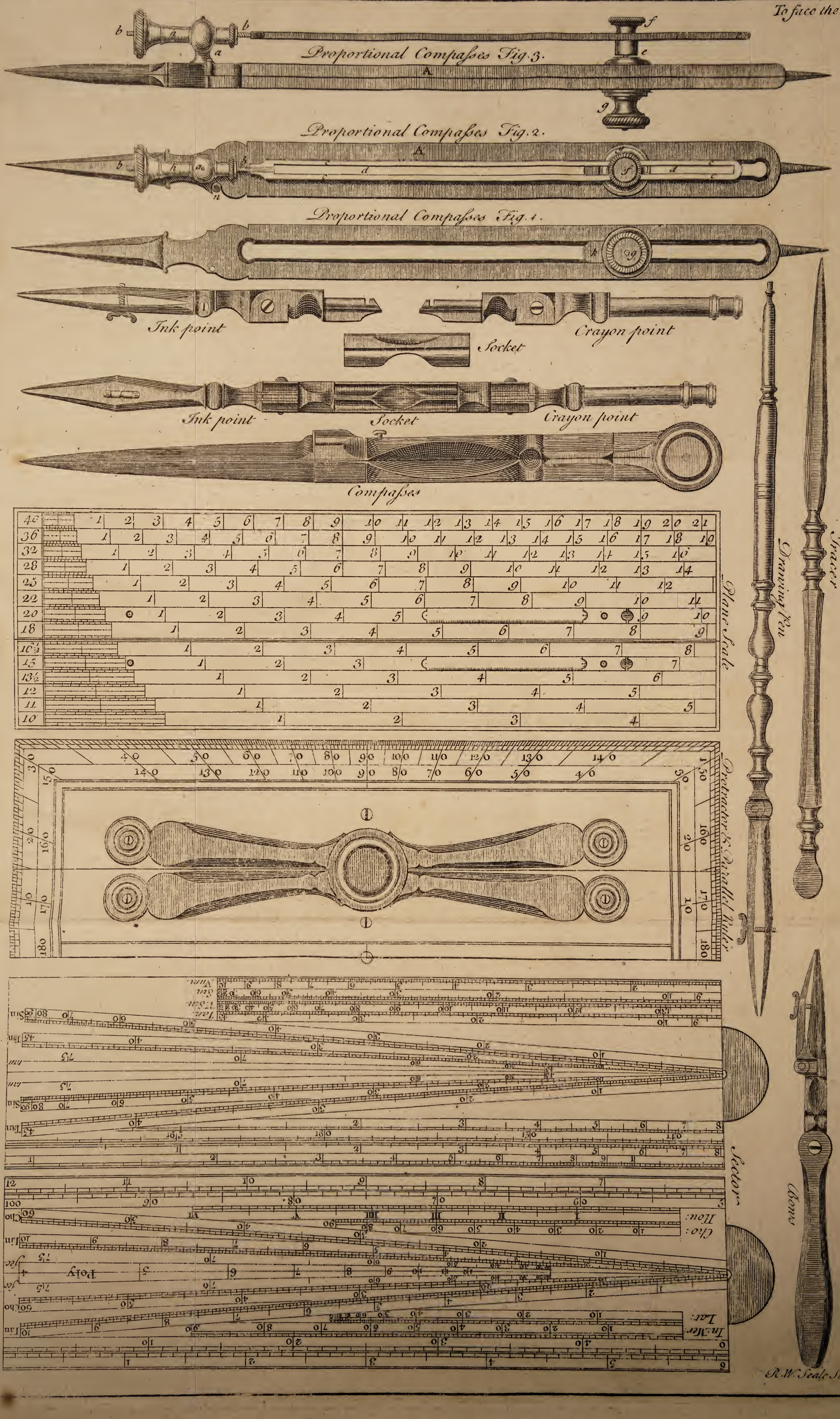
Proportional Compaſses Fig. 3.
Proportional Compaſses Fig. 2.
Proportional Compaſses Fig. 1.
Ink point
Crayon point
Socket
Ink point
Socket
Crayon point
Compaſses
Plane Scale
Protractor & Parallel Ruler
Drawing Pen
Tracer
Sector
Bows
A
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R.W. Seale Sculp.

19101

# A
# TREATISE
# OF SUCH
# Mathematical Inſtruments

As are uſually put into a
PORTABLE CASE,
Containing their various Uſes in

ARITHMETIC,
GEOMETRY,
TRIGONOMETRY,
ARCHITECTURE,
SURVEYING,
GUNNERY, &c.

With a ſhort Account
Of the AUTHORS who have treated on the
PROPORTIONAL COMPASSES
And SECTOR.

To which is now added
An APPENDIX;
Containing, the Deſcription and Uſe of the
GUNNERS CALLIPERS.

The ſecond Edition, with many Additions.

By J. ROBERTSON, F. R. S.
Maſter of the ROYAL-ACADEMY at *Portſmouth*.

*LONDON:*
Printed for T. HEATH and J. NOURSE in the
*Strand*; J. HODGES on *London-Bridge*, and
J FULLER in *Ave-mary-Lane*. MDCCLVII.

# ERRATA.

Page xvi. read March 5, 1757. p. 10. Line 16. for Plate iii read Plate iv. p. 12. l. 15. for KF r. KC. p. 26. l. 26. for into r. in. p. 40. l. 19. for Scale r. Scales. p. 63. l. 6. dele K. l. 29. for B3 r. B5. p. 80. l. 9. r. As the diviſor, is to unity; ſo is the dividend, to the quotient. And as the diviſor, is to the dividend; ſo is unity, to the quotient. p. 81. l. 19. for 25 r. 35. p. 95. l. 23. for xvii r. xviii. p. 96. l. 12. read Ex. 1. Pl. vi. Fig. 26. p. 100. l. 4. for N. r. x. l. 20. for N. r. x. p. 101. l. 34. for N r. x. l. 35. for M r. z. l. 36. for N r. x. p. 107. l. 1. for xviii r. xix. p. 109. l. 17. for on B read on P. p. 110. In the computation for the letter C read D. p. 125. l. 24. for xix read xx.

Page 128. p. 12. for $Ac=\frac{1}{n+1}N$ read $Ac=\frac{n}{n+1}N$.

Line 16 for $\frac{n}{1}$ read $\frac{1}{n}$. p. 145. l. 19. for $\frac{2}{5}$ r. $\frac{2}{3}$. p. 149. l. 24. for balks r. bulks. p. 155. l. 19 for 1. 8. r. 1. 0.

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TO

# *PETER DAVALL*, Eſq;

## SECRETARY to the ROYAL SOCIETY.

SIR,

IT is no new thing for a lover of Science to addreſs his productions to a friend eminently diſtinguiſhed for his general knowledge, as well as particular ſkill in the parts whereon the Author writes: On this account I heartily wiſh, that inſtead of the ſubjects contained in the following ſheets, I had a work of a more elevated kind wherewith to do greater honour to the name of my friend; however, ſuch as they are, I hope they will, with your uſual frankneſs and good-nature, be accepted. Indeed I muſt obſerve, that the late Preſident of the Royal Society, MARTIN FOLKES, Eſq; honoured the firſt Edition of this book with his Patronage; and alſo, our much-eſteemed and learned friend JAMES BURROW, Eſq; Vice-preſident of the Royal Society, thought the book ſo worthy his peruſal, as to remark all the typographical and other errors, and to make ſome uſeful obſervations, a liſt whereof he favoured me with, and for which I truſt you will permit me to take this opportunity of publickly thanking him: Although I am conſcious, that you have the higheſt regard for the two reſpectable names, which I here mention out of gratitude; yet I would not be underſtood that you are to accept hereof in this public manner, merely becauſe thoſe conſiderable perſonages have already favoured the Work; I offer this as a tribute for your acquaintance and friendſhip, and flatter myſelf that you will find in this impreſſion ſome things, which if they have not difficulty to recommend them, have at leaſt, I apprehend, ſo much utility accompanying them, as to render the whole in ſome degree intereſting, and perhaps not unworthy the notice of the moſt ſkilful in the Mathematical Sciences. I am,

SIR,

Your moſt obedient

Humble Servant,

J. ROBERTSON.

# TO THE

# READER.

IT is needleſs to enumerate the many purpoſes, to which *mathematical inſtruments* ſerve; their uſe ſeems quite neceſſary to *perſons* employed in moſt of the active ſtations in life.

The *Architect*, whether *civil*, *military*, or *naval*, never offers to effect any undertaking, before he has firſt made uſe of his *rule* and *compaſſes*; and fixed upon a ſcheme or drawing, which unavoidably requires thoſe *inſtruments*, and others equally neceſſary.

The *Engineer*, cannot well attempt to put in execution any deſign, whether for *defence*, *offence*, *ornament*, *pleaſure*, &c. without firſt laying before his view, the plan of the whole; which is not to be conveniently performed, but by *rulers*, *compaſſes*, &c.

There are indeed, very few, if any good *Artificers*, who have not in ſome meaſure, occaſion for the uſe of one or more *mathematical inſtruments*; and whenever there is required, an accurate drawing of a thing to be executed, or repreſented; that collection of inſtruments, uſually put in *portable caſes*, is then abſolutely neceſſary: And of theſe, the moſt common ones, or others applicable to like ſervice, muſt have been in uſe, ever ſince mankind have had occaſion to provide for the neceſſary conveniencies of life: But the *parallel ruler*, the *proportional compaſſes*, and the *ſector*, are not of any great antiquity.

However, by means of the opportunity, which the author had of conſulting moſt, if not all the principal pieces, that have been wrote on this ſubject †; he thinks it will ſufficiently appear from what follows, who were the inventors of theſe latter inſtruments; and when they were firſt known and made uſe of.

I. *Gaſpar Mordente*, in his book *on the compaſſes*, printed in folio at *Antwerp*, 1584; gives the conſtruction and uſe of an inſtrument, invented by his brother *Fabricius Mordente*, in 1554; and by him preſented to the emperor *Maximilian* II. in 1572: *Fabricius* preſented it afterwards, with ſome improvements, to *Rodolphus* II. the ſon of *Maximilian*: In 1578, *Gaſpar* ſtudied to apply the inſtrument to various uſes by the command of the then governor of the *Netherlands*. The inſtrument conſiſts of two flat legs, moveable round a joint like a common pair of compaſſes; but the ends or points are turned down at right angles to the legs, ſo as to meet in one point when the legs are cloſed. In each leg there is a groove, with a ſlider fitted to it, carrying a perpendicular point; ſo that theſe alſo appear like one point when the legs are cloſed, and the ſliders are oppoſite. This compaſs is jointly uſed with a rod, containing a ſcale of equal parts; whereof 30 are equal to the length of each leg. As the operations with this compaſs, depend on the properties of ſimilar triangles, therefore its principles are the ſame with thoſe of the ſector: And moſt, or all the problems that are performed by the line of lines only, can with almoſt the ſame eaſe, be performed by theſe; the tranſition from this inſtrument to the ſector is very natural and eaſy.

The uſe of this inſtrument, is exemplified in problems concerning lines, ſuperficies, ſolids, and meaſuring of inacceſſible diſtances.

The author, p. 22, ſays, he invented an inſtrument there deſcribed; which is our parallel ruler with parallel bars: The parallel ruler with croſs bars, is a more modern contrivance.

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† In the collection of the late *William Jones*, Eſq;

II. *Daniel Speckle*, in the year 1589, publiſhed in folio, his *military architecture*, at *Straſburg*; where he was architect. In his ſecond chapter, he takes notice of compaſſes then in uſe of a curious invention, whoſe center could be moved forwards or backwards, ſo that by the figures and diviſions mark'd thereon, a right line could be readily and correctly divided into any number of equal parts, not exceeding 20. This inſtrument has been ſince called the *proportional compaſſes*.

In the ſame chapter he mentions another compaſſes, with an immoveable center, and broad legs, whereon were drawn lines proceeding from the center, and divided into equal parts; whereby a right line could be divided into equal parts not exceeding 20; becauſe the diviſions on the lines ſtill kept the ſame proportion, to whatever diſtance the legs were opened. This inſtrument was afterwards call'd the *ſector*.

III. Dr. *Thomas Hood*, printed at *London*, *Anno* 1598, a quarto book, intituled, *The making and uſe of a Geometrical Inſtrument called a Sector*. This inſtrument conſiſts of two flat legs, moveable about a joint; on theſe are ſectoral lines, of equal parts, of polygons, and of ſuperficies; that is, lines ſo diſpoſed, as to make all the operations that depend on ſimilar triangles quite eaſy, and that without the laying down of any figure. To the legs is fitted a circular arc, an index moveable on a joint, and ſights, whereby it is made fit to take angles.

IV. *Chriſtopher Clavius*, in his *practical geometry*, printed in quarto at *Rome*, *Anno* 1604, in page 4, ſhews the conſtruction and uſe of an inſtrument, which he calls the *inſtrument of parts*; it conſiſts of two flat rulers moveable on a joint; on one ſide of theſe legs, are the ſectoral lines of equal parts; on the other ſide, are thoſe of the chords: After ſhewing ſome of their uſes, he concludes with ſaying, he is ſenſible of many others to which it may be applied, but leaves them for the exerciſe of the reader to diſcover.

V. *Levinus Hulſius*, in his book of *mechanical inſtruments*, printed in quarto at *Frankfort*, *Anno* 1605; gives, in the third part, the deſcription and uſe of an inſtrument, which *Juſtus Burgius* call'd the *proportional Compaſs*. *Hulſius* ſays, the uſe of it had not been publiſhed before, although the inſtrument had been long known.

VI. *Anno* 1605, *Philip Horſcher*, M. D. publiſhed at *Mentz*, a quarto book, containing the uſe and conſtruction of the *proportional compaſſes*. This author does not pretend to be the inventor; but that ſeeing ſuch an inſtrument, he thought he could, from *Euclid*, ſhew its conſtruction and the grounds of its operations.

VII. *Anno* 1606, *Galilæus* publiſhed in *Italian*, a treatiſe of the uſe of an inſtrument which he calls, *The geometrical and military compaſs*. On this inſtrument are deſcribed ſectoral lines of equal parts, ſurfaces, ſolids, metals, inſcribed polygons, polygons of given areas, and ſegments of circles. In the preface to an edition of this book, printed at *Padua*, *Anno* 1640, by *Paola Frambotti*, *Galilæus* ſays, that on account of the opportunity he had of teaching mathematics at *Padua*, he thought it proper to ſeek out a method of ſhortening thoſe ſtudies. In another part of the preface he ſays, that he ſhould not have publiſhed this tract, but in vindication of his own reputation; for he was informed that a perſon had by ſome means or other, got one of his inſtruments, and pretended to be the inventor, although himſelf had taught it ever ſince the year 1597.

VIII. *Anno* 1607, *Baldeſſar Capra*, publiſhed a treatiſe of the conſtruction and uſe of the *compaſs of proportion*, (or ſector.) He claims the invention of this inſtrument; and hence aroſe a diſpute between *Galilæus* and *Capra*; ſome particulars of which have been mentioned by ſeveral, and particularly by *Thomas Saluſbury*, Eſq; in his life of *Galilæus*, publiſhed at the end of the ſecond volume of his *mathematical collections and tranſlations*, at *London*, in fol. *Anno* 1664.

It appears from theſe accounts that one *Simon Marius* a German who was in *Padua* about the year 1607, tranſlated into latin, the book publiſhed the year before by *Galilæus*, and cauſed his diſciple *Capra* to print it as his own: *Marius* dreading a proſecution, retired, and left *Capra* in the lurch, who was proceeded againſt. At that time *Galilæus* publiſhed an apology, intitled, "*The defence of* Galilæus Galilæi, *a* Florentine *gentleman, reader in the univerſity of* Padoua, *againſt the calumnies and impoſtures of* Baldeſſar Capra *a* Milaneſe, *divulged againſt him as well in his conſideratione aſtronomica upon the new ſtar of* 1604, *as (and more notoriouſly) in lately publiſhing for his invention the conſtruction and uſes of the geometrical and military compaſs, under the title of Uſus & Fabrica circini cujuſdam proportionis, &c.*" *Galilæus* begins with an addreſs to the reader, wherein he concludes, that a perſon robbed of his inventions, ſuffers the greateſt loſs that can be ſuſtained, becauſe it deſpoileth him of *honour, fame and deſerved glory:*" He proceeds, and ſays, "*into this ultimate of miſeries and unhappineſs of condition,* Baldeſſar Capra, *a* Milaneſe, *with unheard of fraud, and unparallel'd impudence hath endeavoured to reduce me, by lately publiſhing, and committing to the preſs my geometrical and military compaſs, as his proper invention, and as a production of his own wit, (for ſo he calls it in the work itſelf) when it was I alone, that ten years ſince* (viz. *Anno* 1597) *thought of, found and compleated the ſame, ſo as that no one elſe hath any ſhare in it; and I alone from that time forward imparted, diſcovered and preſented it unto many great princes, and other noble lords; and in fine, only that I a year ſince cauſed the operations thereof to be printed, and conſecrated to the glorious name of the moſt ſerene prince of* Tuſcany, *my lord.*

*lord. Of which ſaid inſtrument the above-named* Capra, *hath not only made himſelf the author, but reports me for its ſhameleſs uſurper, (theſe are his very words) and conſequently bound to bluſh within my ſelf with extream confuſion, as unworthy to appear in ſight of learned and ingenuous men.*" *Galilæus* then proceeds, among other things, to produce the atteſtations of four conſiderable perſons, ſhewing that ten years before that time, he had taught the uſe of the inſtrument, and that *Capra* who had for four years paſt ſeen them making at the workman's houſe, had never challenged the invention, as his own.

*Galilæus* after this, ſays that he went to *Venice*, and laid the affair before the lords reformers of the univerſity of *Padoua*, on the 8th of *April* 1607, at the ſame time ſhewing them his own book, publiſhed *June* the 10th 1606; and that of *Capra*'s, publiſhed *March* the 7th 1607. The lords thereupon cited *Capra* to appear before them on the 18th of *April*; the next day the cauſe was heard and the parties diſmiſſed: But on the 4th of *May* following, their lordſhips pronounced ſentence, and ſent it to *Padoua* to be put in execution; the amount of their ſentence was, that having fully conſidered the affair, it appeared to them that *Galilæus* had been abuſed, and that all the remaining copies of *Capra*'s book ſhould be "*brought before their lordſhips to be ſuppreſſed in ſuch faſhion as they ſhall think fit, reſerving to themſelves to proceed againſt the printer and bookſeller, for the tranſgreſſions they may have committed againſt the laws of printing*; *ordering the ſame to be made known accordingly.*

*The ſame day all the copies of the ſaid book were ſent to* Venice *unto the lords reformers*; *there being found* 440 *with the bookſeller, and* 13 *with the author, he having diſtributed* 30 *of them into ſundry parts of Europe*, &c."

IX. *Anno* 1610, *John Remmelin*, M. D. publiſhed at *Frankfort*, a quarto edition of two tracts of *John Faulhaber*; one of theſe contains the uſe of the *ſector*, on which are lines of equal parts, ſuperficies, ſolids, metals, chords, *&c.* He ſays, that *G. Brendel*, a painter, uſed this inſtrument in perſpective painting.

X. *D. Henrion*, in his *mathematical memoirs*, *Anno* 1612, gave a ſhort tract of the uſe of the compaſs of *proportion* (or *ſector.)* In 1616 he printed a book of the uſe of the ſector; and a fifth edition, in the year 1637, the preface to which, ſeems to be wrote in the year 1626, wherein he ſays, that about the year 1608, he had ſeen in the hands of *M. Alleaume*, engineer to the king of *France*, one of theſe ſectors; whereupon he wrote ſome uſes of it, which he publiſhed in his memoirs, as above. He alſo declares, that before his firſt publication, he had not ſeen any book on the uſe of a ſector, and therefore calls what he publiſhes his own. He charges Mr. *Gunter* with having uſed many of his propoſitions. This author printed at *Paris* 1626, an octavo book of logarithms, at the end of which is a tract call’d logocanon, or the proportional ruler; which is a deſcription and uſe of an inſtrument, he calls a lattice, (perhaps from the chequer-work made by lines drawn thereon) which operates the problems performed by the *french* ſectors very accurately.

XI. *Anno* 1615, *Stephen Michael-Spackers*, publiſhed in quarto at *Ulm*, a treatiſe of the *proportional rule and compaſs of G. Galgemeyer*, reviſed by *G. Brendel*, a painter at *Laugingen*. On theſe proportional compaſſes, are lines of equal parts, of polygons, ſuperficies, ſolids, ratio of the diameter to the circumference; reduction of planes, and reduction of ſolids. The uſe and conſtruction of theſe lines, are ſhewn by a great variety of examples.

XII. *Benjamin Bramer*, in his book of *the deſcription of the proportional ruler and parallelogram*, printed in quarto at *Marpurg*, *Anno* 1617; ſays, his ruler is applicable to the ſame uſes as *Juſtus Burgius*'s inſtrument. *Bramer*'s inſtrument conſiſts of a ruler, on which are lines of equal parts, of ſuperficies, of ſolids, of regular ſolids, of circles, of chords, and of equal polygons; at the beginning of each ſcale, is a pin-hole, whereby he can apply the edge of another ruler, and ſo conſtitute a ſector for each ſcale.

XIII. *Anno* 1623, *Adriano Metio Alcmariano*, printed at *Amſterdam* a quarto book, ſhewing the uſe of an inſtrument called the *rule of proportion*. In his dedication, he ſays, that whilſt he was reviewing ſome things relating to practical geometry, he met with *Galileo*'s book of the uſe of the ſector, which gave him opportunity to improve on it, and occaſioned the publiſhing of this book.

XIV. Mr. *Edmund Gunter*, profeſſor of aſtronomy in *Greſham* college, printed at *London*, *Anno* 1624, a quarto book, called *the deſcription and uſe of the ſector*; on which are ſectoral lines, 1ſt. of equal parts; 2d. ſuperficies; 3d. ſolids; 4th. ſines and chords; 5th. tangents; 6th. rhumbs; 7th. ſecants: Alſo lateral lines of, 8th. quadratures; 9th. ſegments; 10th. inſcribed bodies; 11th. equated bodies; 12th. metals: On the edges are a line of inches and a line of tangents.

Mr. *Gunter* does not ſay any thing concerning the invention, and has no preface; but at the end of the tract, in a concluſion to the reader, he ſays, that the ſector was thus contrived, moſt part of the book written, and many copies diſperſed, more than ſixteen years before, *&c.* this article being written *May* 1, 1623, brings the time he ſpeaks of to about the year 1607, which was before the time *Henrion* ſays he firſt ſaw the ſector.

The ſcales of logarithm numbers, ſines, and tangents, were firſt publiſhed in 1624, in *Gunter*'s deſcription of the croſs ſtaff.

XV. *Mutio Oddi* of *Urbino* printed at *Milan*, *An.*

1633, a quarto book, called *the conſtruction and uſe of the compaſſo polimetro*, (or ſector.) The lines on this inſtrument, were ſuch as were common at that time: He ſays in the dedication to his friend *Peter Linder* of *Nurenberg*, he firſt taught the uſe of it.

In the preface he ſays, that about the year 1568, *Commandine*, who then taught at *Urbino*, did contrive a pair of compaſſes with a moveable centre, to divide right lines into equal parts; which was done at the requeſt of a gentleman named *Bartholomew Euſtachio*, who wiſhed to avoid the trouble of the common methods, or of being obliged to have many compaſſes for ſuch diviſions of right lines.

He farther ſays, that about that time, *Guidibaldo*, marqueſs of *Monte*, who lived at *Urbino* for the ſake of *Commandine*'s company, being frequently at the houſe of *Simone Boraccio*, who made *Commandine*'s proportional compaſſes, did contrive, and cauſe to be made, an inſtrument with flat legs, (like the ſector) which performed the operations of the compaſs more eaſily. *Oddi* ſays alſo, that great numbers were made, and in few years, had many uſeful and curious additions, with treatiſes written on its uſe in diverſe languages, and called by different names, which occaſioned the doubt of who was the true author, every one having found means to ſupport his cauſe: But *Oddi* ſays, he not intending to decide the diſpute, leaves it to time to diſcover; and ſeems contented to have pointed out who was the firſt inventor; his chief intention being that of making the uſe public, and the conſtruction eaſy to workmen.

The following authors have alſo wrote on the ſector, and ſectoral lines.

XVI. *Anno* 1634, *P. Petit*, printed in 8vo. at *Paris*, a treatiſe on the ſector. He thinks *Galilæus* was the inventor.

XVII. *An.* 1635, *Matthias Berneggerus* printed at *Straſburg* a 4to. edition of *Galilæus*'s book on the ſector, which conſiſts of two parts: To this is added a third part, ſhewing the conſtruction of *Galilæus*'s lines, and ſome additional uſes and tables.

XVIII. *An.* 1639, *Nicholas Foreſt Duchesſne* printed at *Paris*, in 12mo. a book of the ſector. He ſeems to be little more than a copier of *Henrion.*

XIX. *An.* 1645, *Bettinus* in his *Apiaria univerſa*, &c. *apiar.* 3d. p. 95, and *apiar.* 12, p. 4. In his *Ærarium philo. math.* 4to. *an.* 1648, vol. I. p. 262. In his *Recreationum math. appiariæ*, &c. 12mo. *an.* 1658, p. 75, applies the ſector to muſic.

XX. *John Chatfield* printed at *London*, in 12mo. his *trigonal ſector*, *anno* 1650.

XXI. *An.* 1656, *Nicholas Goldman* printed at *Leyden*, in folio, his *treatiſe on the ſector.* He ſays that *Galilæus* was the firſt who publiſhed the deſcription of the ſector, an invention uſeful in all parts of the mathematics, and other affairs of life.

XXII. *John Collins* printed at *London*, in 4to. his book of *the ſector on a quadrant*, *an.* 1659.

XXIII. *Pietro Ruggiero*, in his *military architecture*, in 4to. printed at *Milan*, *an.* 1661, p. 230, applies the ſector to the practice of fortification.

XXIV. *An.* 1662, *Gaſpar Schottus* printed at *Straſburgh* his *matheſis cæſaræa*, in 4to. in which he gives a deſcription and uſe of the ſector: In the preface he mentions *Galilæo* as the inventor of the ſector.

XXV. *J. Templar* printed in 12mo. at *London*, *an.* 1667, a book called *the ſemicircle on a ſector.* He ſays, the applying of Mr. *Forſter*'s line of verſed ſines to the ſector, was firſt publiſhed *an.* 1660, by *John Brown*, mathematical inſtrument-maker in *London.*

XXVI. *Daniel Schwenter* in his *practical geometry*, reviſed and augmented by *George Andrew Bocklern*, printed in 4to. at *Nuremberg*, *an.* 1667, treats on the deſcription and uſe of the ſector.

XXVII. *John Caramuel* printed at *Campania*, *an.* 1670, his *matheſis nova*, in 2 vols. folio. In the 2d vol. p. 1158, he treats on the ſector, relates the conteſt between *Galilæus* and *Capra*, and thinks the ſame might have been objected againſt others, as well as againſt *Capra:* He alſo ſays, that *Clavius* had ſuch an inſtrument before that of *Galilæus* appeared; and *Clavius* having taught for a long time at *Rome*, had many ſcholars, ſome of whom might have carried his inſtruments to ſeveral countries. *Caramuel* mentions a ſtory of a *Hollander* ſhewing to *Galilæus* an inſtrument of this ſort, that he had brought from his country, and of which *Galilæus* took a copy.

XXVIII. *John Brown*, in his book on *the triangular quadrant*, printed in 8vo. at *London*, *an.* 1671.

XXIX. *John Chriſtopher Rohlhans*, in his *math. and optical curioſities*, printed in 4to. at *Leipſic*, *an.* 1677, p. 216.

XXX. *An.* 1683, *Staniſlawa Solſkiego* printed at *Kracow*, his *geometria et architectura Polſki*, in folio. p. 69, treats on ſome ſectoral lines.

XXXI. *Henrick Jaſper Nuis*, printed at *Tezwolle*, in 4to. his *Rectangulum catholicum geometrico aſtronomicum*, *an.* 1686.

XXXII. *De Chales*, in his *curſus mathem.* printed at *Leyden*, in 2 vols. fol. *an.* 1690. Vol. 2d. p. 58, relates the conteſt between *Galilæus* and *Capra*, and aſcribes the invention of the proportional compaſs to Dr. *Horſcher*, or *Juſtus Burgius*.

XXXIII. *An.* 1691, an edition in 8vo. of Mr. *Ozanam*'s treatiſe of the ſector, was printed at the *Hague*.

XXXIV. *P. Hoſte* printed at *Paris* his courſe of mathematics, in 3 vols. 8vo. *an.* 1692. In vol. 2d. p. 27. he gives a tract on the ſector.

XXXV. *Thomas Allingham* in his *ſhort treatiſe on the ſector*, in 4to. *London*, 1698.

XXXVI. *J. Good*, in his *treatiſe on the ſector*, in 12mo. *London*, 1713.

XXXVII. *Chriſtian Wolfius*, in his *math, lexicon*, 8vo. printed at *Leipſic*, *an.* 1716, under the word *circinus proportionum*, relates, that *Levinus Hulſius*, in his treatiſe on the proportional compaſſes, printed at *Frankfort* the 10th of *May*, 1603, ſays, that he firſt ſaw the ſaid inſtrument at *Ratiſbon*, on the day of the imperial dyet: That he had ſold them far and near before 1603; and that it had been inaccurately copied in ſeveral places: *Wolfius* ſays farther, that *Juſtus Burgius* was certainly the inventor, but uſed to let his inventions lye unpubliſhed.

He then relates the conteſt between *Galilæus* and *Capra*, and ends with ſhewing the difference between the inſtruments of *Burgius* and *Galilæus*.

XXXVIII. *M. Bion*, in his conſtruction of mathematical inſtruments, tranſlated by *Edmund Stone*, fol. *London*, 1723.

XXXIX. Mr. *Belidor*, in his *new courſe of math.* in 4to. p. 364, *Paris*, 1725.

XL. *Roger Rea*, in his *ſector and plane ſcale compared*, 8vo. *London*, 1727, 2d edition.

XLI. *Vincent Toſco*, in his *compendium of the math.* in 9 vols. 8vo. *Madrid*, 1727, vol. I. p. 359.

XLII. *Jacob Leupold*, in his *theatrum arithmetico-geometricum*, in fol. *Leipſic*, 1727. p. 86, gives a detail of the inventors of the proportional compaſſes and ſector, which goes on to p. 121, and then he gives a liſt of the authors who have wrote on proportional inſtruments, *viz. Bramer*, 1617; *Capra*, 1607; *Caſati*, 1664; *Conette*, 1626; *Dechales*, 1690; *Dolz*, 1618; *Faulhaber*, 1610; *Galgemeyer*, 1615; *Brendell*, 1611: *Galilæus*, 1612; *Goldman*, 1656; *Horſcher*, 1605; *Horen*, 1605; *Hulſius*, 1604; *Clavius*, 1615; *Lockmann*, 1626; *Metius*, 1623; *Patridge*—; *de Saxonica*, 1619; *Scheffelts*, 1697; *Steymann*, 1624; *Uttenhoffers*, 1626.

XLIII. *Samuel Cunn*, in his *new treatiſe on the ſector*, 8vo. *London*, 1729.

XLIV. *William Webſter*, in his appendix to a tranſlation of *P. Hoſt*'s mathematics, 8vo. 2 vols. *London*, 1730.

There may be ſeveral other authors who have wrote on the conſtruction and uſe of the ſector, or on ſome of the ſectoral lines; but thoſe above, are all that have come to hand; and indeed theſe are many more than are wanted to determine this enquiry; which may be collected chiefly, from *Mordente*, *Speckle*, *Hood*, *Clavius*, *Hulſius*, *Galilæus*, *Oddi*, *Salluſbury*, *Caramuel*, *Dechales*, *Wolfius*, and *Leupold*; the others ſerving only to inform the reader what works are extant on this ſubject. From the whole he may obſerve, that there are few countries in *Europe*, but have one or more treatiſes on the proportional compaſſes and ſector, in their own language; and this is ſufficient to ſhew, that theſe inſtruments have been in univerſal eſteem.

As the publication of *Mordente*'s book was in 1584, it is not improbable, as *Caramuel* relates, that a *Hollander* (or one from the neighbourhood of *Antwerp*) might ſhew one of *Mordente*'s inſtruments to *Galilæus*: Neither is it improbable that *Galilæus* had ſeen both *Mordente*'s and *Speckle*'s books, the former having been publiſhed thirteen years, and the latter eight years, before *Galilæus*, by his own accounts, thought of his inſtrument.

As *Mutio Oddi*, was a native of *Urbino*, and from what he ſays in his dedication, it is not improbable but he was acquainted with one or more of the perſons he mentions in his preface, or at leaſt with ſome of their acquaintance, from whom he might gather the particulars he relates; to which, if any credit may be given, *Commandine* was the inventor of the proportional compaſſes, and *Guidobaldo* of the ſector: And in the intercourſe between *Italy* and *Germany*, ſome of *Simone Borachio*'s work might get into the hands of many ingenious *Germans*, and give *Juſtus*

*Burgius*, to whom the proportional compaſs is uſually aſcribed, opportunity of getting an early copy; and alſo put into *Speckle*'s way, the inſtrument he mentions to have ſeen: His deſcription pretty nearly agreeing with what *Oddi* ſays was contrived by *Guidobaldo*.

But while we are ſearching among foreigners for the inventor of the ſector, what are we to think of our countryman Dr. *Hood?* who in 1598 publiſhed his account of an inſtrument which he really calls a ſector: And though we ſhould allow that *Hood* as well as *Galilæus* might have ſeen *Mordente*'s and *Speckle*'s books; and both of them might have ſeen ſome of *Borrachio*'s work, yet it is not very probable that *Hood* could have got the form of his inſtrument from *Galilæus* the year after he thought of it; and as *Hood* publiſhed eight years before *Galilæus*, *Hood* certainly has an equal right with *Galilæus*, if not a greater, to the honour of the invention of the ſector.

After all, it may be ſaid, that it is not impoſſible for the ſame thing to be diſcovered by different perſons who have no connexion with one another; examples of a like coincidence of thoughts being known on other ſubjects.

To the preſent edition, there is added an appendix on the gunners callipers, which was promiſed to the public in the former impreſſion, publiſhed at the beginning of the year 1747; and beſide this, the body of the book has been augmented by more than three ſheets of additional illuſtrations and problems, and another plate: By all theſe additions, it is conceived the book is now rendered more generally uſeful.

What is done in the foregoing eſſay, and in the following work, is ſubmitted to the reader's judgment; the author intending no more than to have the honour of invention aſcribed to whom it is due; and alſo to give ſome aſſiſtance to beginners in the mathematical ſtudies.

*Royal Academy Portſmouth*
*March* 5, 1755.

# THE

# CONTENTS.

Section. Page.
I. *OF the common portable inſtruments and caſes.* 1
II. *Of compaſſes.* 3
*Of the bows.* 6
III. *Of the black-lead pencil, feeder, and tracing-point.* 7
*To trace or copy a drawing.* ibid.
IV. *Of the drawing-pen and protracting-pen.* 8
V. *Of the parallel-ruler, and its uſe.* 9
1ſt. *In drawing of parallel right lines.* 10
2d. *In the dividing of right lines into equal parts.*
3d. *In the reduction of right-lined figures to right-lined triangles of equal area.* 11
VI. *Of the protractor, and its uſe.* 13
1ſt. *In plotting and meaſuring of right-lined angles*; 14
2d. *In drawing of right lines perpendicular to each other*;
3d. *In inſcribing of regular polygons in a circle*; 15
4th. *In deſcribing of regular polygons on given right lines.* 16
VII. *Of the plane ſcale, and its ſeveral lines.* 18
*Conſtruction of the ſcales of equal parts.* ibid.
*Their uſe, joined with the protractor, in plotting of right-lined figures.* 22
*Conſtruction of the other lines of the plane ſcale,* viz. 1ſt. *Chords*; 2d. *Rhumbs*; 3d. *Sines*; 4th. *Tangents*; 5th. *Secants*; 6th. *Half Tangents*; 7th. *Longitude*; 8th. *Latitude*; 9th. *Hours*; 10th. *Inclination of Meridians.* 23

Sect. Page.

VIII. *The uſes of ſome of the lines on the plane ſcale.* 27

*A table, ſhewing the miles in one degree of longitude to every degree of latitude.* 29

IX. *Of the ſector and its lines.* 30

X. *Of the conſtruction of the ſingle ſcales on the ſector.* 33

XI. *Of the conſtruction of the double ſcales on the ſector.* 37

XII. *Of the uſes of the double ſcales.* 40

*The uſe of the lines of lines.*

1ſt. *To two right lines given, to find a* 3d *proportional.* 41

2d. *To three right lines given, to find a* 4th *proportional.* 42

3d. *To ſet the ſcales of lines at right angles to one another.* 43

4th. *Between two right lines to find a mean proportional.* ibid.

5th. *To divide a right line into equal parts.* 44

6th. *To delineate the orders of architecture.* 45

*Some terms in architecture explained.* ibid.

*Of the general proportions in each order.* 47

*To draw the mouldings in architecture.* 55

*Table for deſcribling the Ionic volute.* 59

*Uſes of ſome tables for drawing the orders.* 60

*To delineate any order by the tables.* 62

*Three tables, ſhewing the altitudes and projections, of every moulding and part in the pedeſtals, columns, and entablatures of each order; according to the proportions given by* Palladio.

XIII. *Some uſes of the ſcales of polygons.* 72

XIV. *Some uſes of the ſcales of chords.* 73

*To delineate the ſtation lines of a ſurvey.* 75

XV. *Some uſes of the logarithmic ſcales of numbers.* 79

XVI. *Some uſes of the ſcales of logarithmic ſines, and logarithmic tangents.* 84

XVII. *Some uſes of the double ſcales of ſines, tangents, and ſecants.* 85

*To*

Sect. Page.

*To find the length of the radius to a given ſine, tangent or ſecant.* 88
*To find the degrees correſponding to a given ſine, tangent or ſecant.* 89
*To a given number of degrees, to find the length of the verſed ſine.* ibid.
*To ſet the double lines to any given angle.* ibid.
*To deſcribe an Ellipſis.* 90
*To deſcribe a Parabola.* 91
*To deſcribe an hyperbola.* 92
*To find the diſtance of places on the terreſtrial globe.* 93

XVIII. *The uſe of ſome of the ſingle and double ſcales on the ſector, applied in the ſolution of all the caſes of plane trigonometry.* 95
CASE I. *When among the things given, there be a ſide and its oppoſite angle.* 96
CASE II. *When two ſides and the included angle are known.* 99
CASE III. *When the three ſides are known.* 103

XIX. *The conſtruction of the ſeveral caſes of ſpherical triangles, by the ſcales on the ſector.* 107
CASE I. *Given two ſides, and an angle oppoſite to one of them.* 108
CALE II. *Given two angles, and a ſide oppoſite to one of them.* 112
CASE III. *Given two ſides, and the included angle.* 115
CASE IV. *Given two angles, and the included ſide.* 118
CASE V. *Given the three ſides.* 122
CASE VI. *Given the three angles.* 124

XX. *Of the proportional compaſſes.* 125

*The figures referred to, are contained in ſeven copper-plates.*

## APPENDIX.

Page.
*Of the callipers, and what they contain.* 132
ART. I. *Of the meaſures of convex diameters.* 134
II. *Of the weights of iron ſhot.* 135
III. *Of the meaſures of concave diameters.* 136
IV. *Of the weights of ſhot to given gun bores.* 137
V. *Of the degrees in the circular head.* 138
VI. *Of the proportion of troy and averd. weights.* 139
VII. *Of the proportion of Engliſh and French feet and pounds.* 141
VIII. *Factors uſeful in circular and ſpherical figures.* 142
IX. *Of the ſpecific gravities and weights of bodies.* 147
*Some uſes of the table.* 151
X. *Of the quantity of powder uſed in firing of cannon.* 154
XI. *Of the number of ſhot or ſhells in a finiſhed pile.* 157
XII. *Concerning the fall of heavy bodies.* 161
XIII. *Rules for the raiſing of water.* 164
XIV. *Of the ſhooting in cannon and mortars.* 167
XV. *Of the line of Inches.* 174
XVI. *Of the logarithmic ſcales of numbers, ſines, verſed ſines and tangents.* ibid.
XVII. *Of the line of lines.* 175
XVIII. *Of the lines of plans or ſuperficies.* ibid.
XIX. *Of the line of ſolids.* 180
*Nine Plates.*

To the Binder.

The plates are all to ſtand upright in the book, and no part to be folded upwards or downwards.

# THE
# DESCRIPTION *and* USE
# OF A
# CASE,
# OR
# PORTABLE COLLECTION,
# Of the moſt Neceſſary
# Mathematical Inſtruments.

## SECT. I.

CASES of *Mathematical Inſtruments* are of various ſorts and ſizes; and are commonly adapted to the fancy or occaſion of the perſons who buy them.

THE ſmalleſt collection put into a caſe, commonly conſiſts of,

I. A *pair of compaſſes*, one of whoſe points may be taken off, and its place ſupplied with, A *crayon* for lead or chalks.

A *drawing-pen* for ink.

II. A *plane ſcale.*

WITH theſe inſtruments only a tolerable ſhift may be made to draw moſt mathematical figures.

BUT in ſets, called *complete pocket-caſes*, beſide the inſtruments above, are the following.

III. A ſmaller pair of *compaſſes.*

IV. A pair of *bows.*

V. A *black-lead pencil*, with a *cap* and *feeder.*

VI. A *drawing-pen* with a *protracting-pin.*

VII. A *protractor.*

VIII. A *parallel-ruler.*

IX. A *ſector.*

IN ſome caſes, the plane ſcale, protractor, and parallel-ruler, are included in one inſtrument.

THE common, and moſt eſteemed ſize of theſe inſtruments, is ſix inches; though they are ſometimes made of other ſizes, and particularly of four inches and a half.

*Note*, the ſize of a caſe is named from the length of the ſcale or ſector.

SOME artiſts have contrived a very commodious flat caſe, or box, where the inſide of the lid or top contains the rulers and ſcales: The compaſſes, drawing-pen, &c. lie in the partitions of a drawer, that drops into the bottom part of the caſe, but not quite to the bottom; leaving room under it for *black-lead pencils*, *hair pencils*, *Indian ink*, *colour cells*, &c. and beſide the inſtruments already enumerated, in boxes or caſes of this ſort are put

X. A *tracing-point.*

XI. A pair of *proportional compaſſes.*

XII. A *gunner's callipers.*

BUT the caſe of inſtruments called the *magazine*, is the moſt complete collection; for this contains whatever can be of uſe in the practice of *drawing*, *deſigning*, &c. and as the greateſt part of theſe inſtruments are ſcarcely ever uſed but in the ſtudies or chambers of thoſe who have occaſion for them; therefore it will be uſeleſs to inſiſt on pocket caſes; for few perſons care to load themſelves with the carriage of what is called a *complete ſet*.

## SECT. II.

## *Of the* COMPASSES *and* BOWS.

COMPASSES are uſually made of ſilver or braſs, and thoſe are reckoned the beſt, part of whoſe joint is ſteel; and where the *pin* or *axle* on which the joint turns, is a ſteel ſcrew; for the oppoſition of the metals makes them wear more equable: and by means of the ſcrew axle, with the help of a *turn-ſcrew*, (which ſhould have a place in the caſe) the compaſſes can be made to move in the joint, ſtiffer or eaſier, at pleaſure. If this motion is not uniformly ſmooth, it renders the inſtrument leſs accurate in uſe. Their points ſhould be of ſteel, and pretty well hardened, elſe in taking meaſures off the ſcales, they will bend, or be ſoon blunted. They alſo ſhould be well poliſhed, whereby they will be preſerved free from ruſt a long time.

To one point of the ſmaller compaſſes, it is common to fix in the ſhank a ſpring, which by means of a ſcrew, moves the point; ſo that when the compaſs is opened nearly to a required diſtance, by the help of the ſcrew the points may be ſet exactly to that diſtance; which cannot be done ſo well by the motion in the joint.

### *To uſe the ſpring point.*

HOLD the compaſſes in the left hand with the ſcrew turned towards the right; turn the ſcrew towards you, or ſlacken it, and the ſpring point will be brought nearer to the other point: On the contrary, by turning the ſcrew from you, or tightning it, the ſpring point will be ſet farther from the other point.

The uſe of theſe leſſer compaſſes, is to transfer the meaſures of diſtances from one place to another; or, to deſcribe obſcure arcs.

Of the large ſized compaſſes, thoſe are eſteemed the beſt, whoſe moveable points are locked in by a ſpring and catch fixed in the ſhank; for if this ſpring be well effected, the point is thereby kept tight and ſteady; the contrary of which frequently happens, when the point is kept in by a ſcrew in the ſhank.

The uſe of theſe compaſſes is to deſcribe arcs or circumferences with given radius's: and it is eaſy to conceive, that theſe arcs or circumferences can be deſcribed, either obſcurely by the ſteel point; in ink, by the ink point; in black-lead or chalks, by the crayon; and with dots, by the dotting-wheel; for either of them may be fixed in the ſhank in the place of the ſteel point.

As the dotting-wheel has not hitherto been effected, ſo as to deſcribe dotted lines or arcs, with any tolerable degree of accuracy, it ſeems therefore to be uſeleſs: and, indeed, dotted lines of any kind are much better made by the drawing-pen.

The drawing-pen point, and crayon, have generally (in the beſt ſort of caſes) a ſocket fitted to them: ſo that they occupy but one of the holes, or partitions, in the caſe.

The ink, and crayon points, have a joint in them, juſt under that part which locks into the ſhank of the compaſſes; becauſe the part below the joint ſhould ſtand perpendicular to the plane on which the lines are deſcribed, when the compaſs is opened.

If inſtead of the larger compaſs being made with ſhifting points, there were two pair put into the caſe; to one of which the ink point was fixed, and to the other the crayon point; this would ſave the trouble of changing the points in the compaſs at every time they were uſed; and would increaſe the expence, or bulk of the caſe, but a trifle.

Most perſons at firſt, handle a pair of compaſſes very aukwardly, whether in the taking of diſtances between the points, or deſcribing of circles. To be ſure long practice brings on eaſy habits in the uſe of things, however a caution or two may be ſerviceable to beginners.

### *To open and work the compaſſes.*

With the thumb and middle finger of the right hand pinch the compaſſes in the hollow part of the ſhank, and it will open a little way; then the third finger being applied to the inſide of the neareſt leg, and the nail of the middle finger acting againſt the fartheſt, will open the compaſſes far enough to introduce the fingers between the legs: then the hither one being held by the thumb and third finger, the farther leg may be moved forwards and backwards very eaſily by the fore and middle fingers, the fore finger preſſing on the outſide to ſhut, and the middle one acting on the inſide to open, the compaſſes to any deſired extent. In this manner the compaſſes are manageable with one hand, which is convenient when the other hand is holding a ruler or other inſtrument.

### *To take a diſtance between the points of the compaſſes.*

Hold the compaſſes upright, ſet one point on one end of the diſtance to be taken, there let it reſt; and (as before ſhewn) extend the other point to the other end.

Always take care to avoid working the compaſſes with both hands at once; and never uſe them otherwiſe than nearly upright.

### *To deſcribe circles or arcs with the compaſſes.*

SET one foot of the compaſſes on the point deſigned for the centre, hold the head between the thumb and middle finger, and let the fore finger reſt on the head, but not to preſs it: then by rolling the head between the finger and thumb, and at the ſame time touching the paper with the other point, a circle or arc may be deſcribed with great eaſe, either in lead or ink.

IN deſcribing of arcs it ſhould be obſerved, that the paper be not preſſed at the centre, or under the foot, with more weight than that of the compaſſes; for thereby the great holes and blots may be avoided, which too frequently deface figures when they are made by thoſe who are aukward or careleſs in the uſe of their inſtruments.

### *Of the* BOWS.

The bows are a ſmall ſort of compaſſes, that commonly ſhut into a hoop, which ſerves as a handle to them. Their uſe is to deſcribe arcs, or the circumferences of circles, whoſe radius's are very ſmall, and could not be done near ſo well by larger compaſſes.

## SECT. III.

## *Of the Black-lead Pencil, Feeder, and Tracing Point.*

THE *Black-lead Pencil* is uſeful to deſcribe the firſt draught of a drawing, before it is marked with ink; becauſe any falſe ſtrokes, or ſuperfluous lines, may be rubb'd out with a handkerchief or piece of bread.

The *Feeder* is a thin flat piece of metal, and is ſometimes fixed to a cap that ſlips on the top of the pencil, and ſerves either to put ink between the blades of the drawing-pen, or to paſs it between the points, when the ink by drying, does not flow freely.

The *Tracing Point* is a pointed piece of ſteel; and commonly has the feeder fixed to the other end of the handle. Its uſe, is to mark out the outlines of a drawing or print when an exact copy thereof is wanted, which may be done as follows.

On a piece of paper, large enough to cover the thing to be copied, let there be ſtrewn the ſcrapings of *red chalk*, or of *black chalk*, or of *black lead*; rub theſe on the paper, ſo that it be uniformly covered; and wipe off, with a piece of muſlin, as much as will come away with gentle rubbing. Lay the coloured ſide of this paper, next to the vellum, paper, *&c.* on which the drawing is to be made: on the back of the colour'd paper, lay the drawing, *&c.* to be copied. Secure all the corners with weights, or pins, that the papers may not ſlip: trace the lines of the thing to be copied, with the *tracing point*; and the lines ſo traced will be impreſs'd on the clean paper.

And thus, with care, may a drawing or print, be copied without being much damaged.

*Note*, The coloured paper will ſerve a great many times.

There is not perhaps, a more uſeful inſtrument in being for ready ſervice in making of ſketches or finiſhed plans; whether of architecture, fortification, machines, landſkips, ornaments, *&c.* than a black-lead pencil; and therefore it may be proper to give a few hints concerning this excellent mineral.

Black-lead is produced in many countries, but the beſt yet diſcovered is found in the north of England: it is dug out of the ground in lumps, and ſawed out into ſcantlings proper for uſe: the kinds moſt proper to uſe on paper muſt be of an uniform texture, which is diſcoverable by paring a piece to a point with a penknife; for if it cuts ſmooth and free from hard flinty particles, and will bear a fine point, it may be pronounced good.

THERE are three ſorts of good black-lead; the ſoft, the midling, and the hard: the ſoft is fitteſt for taking of rough ſketches, the midling for drawing of landſkip and ornaments, and the hard for drawing of lines in mathematical figures, fortification, architecture, *&c.* The indifferent kinds, or thoſe which in cutting are found flinty, are uſeful enough to carpenters or ſuch artificers who draw lines on wood, &c.

THE beſt way of fitting black-lead for uſe, is firſt to ſaw it into long ſlips about the ſize of a crow-quill, and then fix it in a caſe of ſoft wood, generally cedar, of about the ſize of a gooſe-quill, or larger; and this caſe is cut away with the lead as it is uſed.

## SECT. IV.

## *Of the Drawing-Pen, and Protracting-Pin.*

THE *Drawing-pen* is an inſtrument uſed only for drawing of right lines; and conſiſts of two blades, with ſteel points, fix'd to a handle. The blades by being a little bent, cauſe the ſteel points to come nearly together; but by means of a ſcrew paſſing thro' both of them, they are brought cloſer at pleaſure, as the line to be drawn ſhould be ſtronger or finer.

IN uſing this inſtrument, put the ink between the blades with a common *pen*, or with the *feeder*; and (by the ſcrew) bring them to a proper diſtance for drawing the intended line: hold the *pen* a little inclined,

clined, but ſo that both blades touch the paper; then may a line be drawn very ſmooth, and of equal breadth, which could not be done ſo well with a common *pen*.

*Note*, BEFORE the *drawing-pen* is put into the caſe, the ink ſhould be wiped from between the blades; otherwiſe they will ſoon ruſt and ſpoil, eſpecially with common ink. And that they may be clean'd eaſily, one of the blades ſhould move on a joint.

THE directions given about this *drawing-pen*, will ſerve for the *drawing-pen* point, uſed with the compaſſes. But it muſt be obſerved, that when any arc is deſcribed of more than an inch radius, then the ink point ſhould be bent in the joint ſo that both the blades of the pen touch the paper, otherwiſe the arc deſcribed will not be ſmooth.

THE *Protracting-pin* is a piece of pointed ſteel (like the point of a needle) fixed into one end of a part of the handle of the *drawing-pen*; into which, the piece with the *pin* in it, generally ſcrews. Its uſe is to point out the interſections of lines; and to mark off the diviſions of the protractor, as hereafter directed.

SOMETIMES on the top of the *drawing-pen* is a ſocket, into which a piece of black-lead pencil may be put.

## SECT. V.

## *Of the* PARALLEL-RULER.

THIS inſtrument conſiſts of two *Rulers*, connected together by two metal bars, moving eaſily round the rivets which faſten their ends; theſe bars are ſo placed that both have the ſame inclination to each *Ruler*; whereby they will be *Parallel* at every diſtance, to which the bars will ſuffer them to receed.

But the beſt *Parallel-Rulers* are thoſe, whoſe bars croſs each other, and turn on a joint at their interſection; one end of each bar moving on a centre, and the other ends ſliding in grooves as the *Rulers* receed.

This inſtrument is very uſeful in delineating civil and military architecture, where there are many *Parallel* lines to be drawn; and alſo in the ſolution of ſeveral geometrical *Problems*; ſome of which are as follows.

## PROBLEM I.

*A right line* AB *being given, to draw a line parallel thereto, that ſhall paſs through a given point* C (Fig. 1. Pl. III.)

Construction. Apply one edge of the *parallel-ruler* to the given line AB; preſs one *ruler* tight againſt the paper, and move the other untill its edge cuts the point C; there ſtay that *ruler*, and by its edge draw a line through C, then this line will be *parallel* to AB.

If the point C happens to be farther from the line AB, than the *rulers* will open to; ſtay that *ruler* neareſt to C, and bring the other cloſe to it, where let it reſt, and move forward the *ruler* neareſt to C, and ſo continue till one *ruler* is brought to the point intended.

The manner of uſing the *parallel-ruler* as here directed, is underſtood to be the ſame in the ſolution of the following Problems.

## PROBLEM II.

*A right line* AB *being given, to divide it into any propos'd number of equal parts; ſuppoſe* 5. (Fig. 2.)

Construction. Draw the indefinite right line BC, ſo as to make with AB, any angle at pleaſure; with any convenient opening of the compaſſes, lay off on BC, the required number of equal parts, *viz.* 1, 2, 3, 4, 5; lay the edge of the *parallel-ruler* by the points 5 and A, and *parallel* thereto, through the points 4, 3, 2, 1, draw lines; then AB, by the interſection of thoſe lines will be divided into 5 equal parts.

## PROBLEM III.

*Any right lined quadrangle or polygon being given, to make a right lin'd triangle of equal area.*

EXAM. I. *To make a triangle of equal area to the quadrilateral* ABDC. (Fig. 3.)

CONSTRUCTION. Prolong AB; draw CB; and through D, draw DE *parallel* to CD, cutting AE in E; then a line drawn from C to E forms the triangle ACE, of equal area to the quadrangle ABDC.

EXAM. II. *Given the pentagon* ABCDE; *requir'd to make a triangle of equal area.* (Fig. 4.)

CONSTRUCTION. Produce DC towards F; draw AC; through B, and *parallel* to AC draw BF cutting DC in F; and draw AF. Then the area of the trapezium AFDE will be equal to the area of the *pentagon* ABCDE.

*Again.* Produce ED towards G; draw AD; through F, draw FG *parallel* to AD, and draw AG. Then the area of the triangle AGE, will be equal to that of the trapezium AFDE; and conſequently, to that of the pentagon ABCDE.

EXAM. III. *To make a triangle equal in area to the Hexagon,* ABCDEF. (Fig. 5.)

CONSTRUCTION. Draw FD, and *parallel* thereto, through E, draw EG meeting CD produced in G, and draw GF. Then the triangle FGD is equal to the triangle FED, and the given *Hexagon* is reduced to the *Pentagon* ABCGF equal in area.

*Again.* Draw AG; through F, draw FH *parallel* to AG, meeting CG produced in H; draw AH, and the *pentagon* is reduced to the trapezium ABCH.

*Laſtly*, Draw AC, and *parallel* thereto, through H, draw HI, meeting BC produced in I, and draw AI. Then the trapezium is reduced to the triangle ABI, which is equal in area to the given *Hexagon* ABCDEF.

EXAM. IV. *Given the nine ſided figure* ABCDEFGHI, *to make a triangle of equal area.* (Fig. 6.)

CONSTRUCTION. 1ſt, Draw IB, and through A draw AK *parallel* to IB, meeting HI produced in K, and draw BK; ſo the three ſides HI, IA, AB, are reduced to the two ſides HK, KB.

2d, Draw KC, and through B draw BL *parallel* to KF, meeting CD in L; draw KL, and the three ſides DC, CB, BK, are reduced to the two ſides DL, LK.

3d, Draw KG; through H, draw HM, *parallel* to KG, meeting GF in M, and draw KM; ſo the three ſides KH, HG, GF, are reduced to the ſides KM, and MF.

4th, Draw KF; through M, draw MN, *parallel* to KE, meeting FE in N, and draw KN; ſo the three ſides KM, MF, FE, are reduced to two ſides KN, NE.

5th, Draw LN, and through K, draw KO, *parallel* to LN, meeting EF produced in O, and draw LO; ſo the three ſides EN, NK, KL, are reduced to the two ſides EO, OL.

*Laſtly*, Draw LE, and through D, draw DP *parallel* to LE, meeting OE produced in P, and draw LP; ſo ſhall the triangle OLP be equal in area to the given nine ſided figure.

PROCEEDING in the ſame manner; a figure of any number of ſides may be reduced to a triangle of equal area.

## SECT. VI.

## Of the PROTRACTOR.

THE *Protractor*, is an inſtrument of a ſemicircular form; being terminated by a right line repreſenting the diameter of a circle, and a curve line of half the circumference of the ſame circle. As at Fig. 7. The point C, (the middle of AB) is the centre of the ſemicircumference ADB, which ſemicircumference is divided into 180 equal parts call'd degrees; and for the convenience of reckoning both ways, is numbered from the left hand towards the right, and from the right hand towards the left, with 10, 20, 30, 40, &c. to 180, being the half of 360, the degrees in a whole circumference. The uſe of this inſtrument is to *protract*, or lay down an angle of any number of degrees, and to find the number of degrees contained in any given angle.

BUT this inſtrument is made much more commodious, by transferring the diviſions on the ſemicircumference, to the edge of a *ruler*, whoſe ſide EF is *parallel* to AB; (ſee Fig. 7.) which is done by laying a *ruler* on the centre C, and the ſeveral diviſions on the ſemicircumference ADB, and marking the interſections of that *ruler* on the line EF, which may eaſily be conceiv'd by obſerving the lines drawn from the centre C to the diviſions 90, 60, 30; ſo that a *ruler* with theſe diviſions mark'd on 3 of its ſides and numbered both ways, as in the *Protractor*, (the fourth or blank ſide repreſenting the diameter of the circle) is of the ſame uſe as a *Protractor*, and is much better adapted to a caſe.

THAT ſide of the inſtrument on which the diviſions are mark'd, is call'd the graduated ſide, or limb of the inſtrument, which ſhould be ſloped away to an edge, whereby the diviſions on the limb will be much eaſier pointed off.

## PROBLEM IV.

*A number of degrees being given; to protract, or lay down an angle whoſe meaſure ſhall be equal thereto. And an angle being protracted, or laid down, to find what number of degrees meaſures that angle.*

Exam. I. *To draw a line from the point* A, *that ſhall make an angle with the line* AB *of* 48 *deg.* Fig. 8.

Apply the blank edge of the protractor to the line AB, ſo that the middle or centre thereof (which is always mark'd) may fall on the point A; then with the protracting-pin, make a mark on the paper againſt the diviſion on the limb of the inſtrument numbered with the degrees given; (*viz.* 48.) counting from the right hand towards the left; a line drawn from A, through the ſaid mark, as AC, ſhall with AB, form the angle required, *viz.* 48 degrees.

If the line had been to make an angle with AB, at the point B; then the centre muſt have been laid on B, and the diviſions counted from the left hand towards the right.

Exam. II. *To find the number of degrees which meaſure the angle* ABC. Fig. 9.

Apply the blank edge of the protractor to the line AB, ſo that the centre ſhall fall on the point B; then will the line BC cut the limb of the inſtrument in the number expreſſing the degrees that meaſure the given angle; which in this *example* is 125 degrees, counting from the left hand towards the right.

## PROBLEM V.

*From any given point* A, *in a line* AB, *to draw a line perpendicular to* AB. Fig. 10.

LAY the protractor acroſs the line AB in ſuch a manner that the centre on the blank edge, and the diviſion numbered with 90, on the limb, may both be cut by the given line; then keeping the ruler in this poſition, ſlide it along the line, till one of theſe points touch the given point A, draw the line CA, and it will be perpendicular to AB.

IN the ſame manner, a line may be drawn, perpendicular to a given line, from a given point out of that line.

## PROBLEM VI.

*In a circle given to inſcribe any regular Polygon. ſuppoſe an octagon.* Fig. 11.

CONSTRUCTION. Apply the blank edge of the *protractor* to AB the diameter of the *Circle*, ſo that their centres ſhall coincide; ſet off a number of degrees from B to D equal to an angle at the centre of that *polygon*, (viz. 45.) and through that mark draw a radius CD; then ſhall BD the chord of the arc expreſſing thoſe degrees, be the ſide of the intended *polygon*; which chord taken between the compaſſes, and applied to the circumference will divide it into as many equal parts as the *polygon* has ſides, *viz.* 8; and the ſeveral chords being drawn will form the *polygon* required.

IT will rarely happen that this operation, though true in theory, will give the ſide of the *polygon* exact; for when the chord of the arc prickt off from the protractor, is taken with the compaſſes and applied to the circle, it generally falls beyond, or ſhort, of the point ſet out from: for it muſt be obſerved that the point where two lines interſect one another is not to be readily determined in a practical manner; and a very ſmall error in the taking the length of the chord, being ſeveral times repeated becomes conſiderable at laſt. Here the compaſſes with the ſpring point will be found of great uſe.

### *A* TABLE, *ſhewing the Angles at the Centres and Circumferences of regular* Polygons *from three to twelve Sides incluſive.*

| Names. | Sides. | Angles at Center | Angles at Cir. |
|---|---|---|---|
| Trigon | 3 | 120° 00′ | 60° 00′ |
| Square | 4 | 90 00 | 90 00 |
| Pentagon | 5 | 72 00 | 108 00 |
| Hexagon | 6 | 60 00 | 120 00 |
| Heptagon | 7 | 51 25$\frac{5}{7}$ | 128 34$\frac{2}{7}$ |
| Octagon | 8 | 45 00 | 135 00 |
| Nonagon | 9 | 40 00 | 140 00 |
| Decagon | 10 | 36 00 | 144 00 |
| Endecagon | 11 | 32 43$\frac{7}{11}$ | 147 16$\frac{4}{11}$ |
| Dodecagon | 12 | 30 00 | 150 00 |

THIS table is conſtructed, by dividing 360, the degrees in a circumference, by the number of ſides in each polygon; and the quotients are the angles at the centers; the angle at the center ſubſtracted from 180 degrees, leaves the angle at the circumference.

## PROBLEM VII.

*Upon a given right line* AB, *to deſcribe any regular polygon.* Fig. 12.

CONSTRUCTION. From the ends of the given line, draw the lines AD, BC; ſo that the angles BAD, ABC, may each be equal to the angle at the circumference in that polygon; make AD, BC, each equal to AB; from the points D and C, draw lines that ſhall make with DA, CB, angles equal to the former; make theſe lines each equal to AB; and ſo continue, till a polygon is form'd of as many ſides as required.

EXAM. I. *Upon the line* AB *to deſcribe an hexagon.* Fig. 12.

DRAW AD, BC, ſo that the angles BAD, ABC, may be each 120 degrees; make AD, BC, each equal to AB: alſo, make the angles ADF, BCE, each equal to 120 degrees, and make DF, CE, each equal to AB; draw FE and 'tis done.

OR it may be done by the help of the parallel ruler, when the polygon has an even number of ſides, Thus,

HAVING form'd the three ſides AD, AB, BC, as before directed; through D, draw DF parallel to BC; make DF equal to AB; through F draw FE parallel to AB: make FE equal to AB and join CE.

EXAM. II. *Upon the line* AB *to deſcribe a pentagon,* Fig. 13.

DRAW AC, BD, that each may make with AB, an angle of 108 degrees. Make AC, BD, each equal to AB; on the points C and D, with the compaſſes opened to the diſtance AB, deſcribe arcs to croſs each other in E; draw EC and ED, and 'tis done.

IN any regular polygon, having found all the ſides but two, as above directed; thoſe may be found as the laſt two in the pentagon were.

BUT a regular polygon deſcribed upon a given line AB may be conſtructed with more accuracy, thus. See Fig. 12, 13.

Make an angle BAP, and another ABP, each equal to half the angle of the required polygon; on the point P, where the lines AP, BP, cut one another, and with the radius PA deſcribe a circle, in which if the given line AB be applied, the polygon ſought will be formed.

## SECT. VII.

### *Of the Plain Scale.*

THE lines generally drawn on the plane ſcale, are theſe following:

| | | Marked |
|---|---|---|
| I. | Lines of equal parts. | E P. |
| II. | ——— Chords. | Cho. |
| III. | ——— Rhumbs. | Ru. |
| IV. | ——— Sines. | Sin. |
| V. | ——— Tangents. | Tan. |
| VI. | ——— Secants. | Sec. |
| VII. | ——— Half Tangents. | S.T. |
| VIII. | ——— Longitude. | Lon. |
| IX. | ——— Latitude. | Lat. |
| X. | ——— Hours. | Ho. |
| XI. | ——— Inclinations. | In. Mer. |

### *Of the Lines of equal Parts.*

LINES of equal parts are of two ſorts, *viz.* ſimply divided, and diagonally divided. Pl. V.

1. *Simply divided.* Draw 3 lines parallel to one another, at unequal diſtances, (Fig. 14.) and of any convenient length; divide this length into what number of equal parts is thought neceſſary, allowing ſome certain number of theſe parts to an inch, ſuch as 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, &c. which diviſions diſtinguiſh by

lines drawn acroſs the three parallels. Divide the left hand diviſion into 10 equal parts, which diſtinguiſh by lines drawn acroſs the lower parallels only; but, for diſtinction ſake, let the 5th diviſion be ſomewhat longer than the others: and it may not be inconvenient to divide the ſame left-hand diviſion into 12 equal parts, which are laid down on the upper parallel line, having the 3d, 6th, and 9th diviſions diſtinguiſhed by longer ſtrokes than the reſt, whereof that at the 6th diviſion make the longeſt.

THERE are, for the moſt part, ſeveral of theſe ſimply divided ſcales put on rulers one above the other, with numbers on the left hand, ſhewing in each ſcale, how many equal parts an inch is divided into; ſuch as 20, 25, 30, 35, 40, 45, *&c.* and are ſeverally uſed, as the plan to be expreſſed ſhould be larger or ſmaller.

THE uſe of theſe lines of equal parts, is to lay down any line expreſſed by a number of two places or denominations, whether decimally, or duodecimally divided; as leagues, miles, chains, poles, yards, feet, inches, *&c.* and their tenth parts, or twelfth parts: thus, if each of the diviſions be reckoned 1, as 1 league, mile, chain, *&c.* then each of the ſubdiviſions will expreſs $\frac{1}{10}$ part thereof; and if each of the large diviſions be called 10, then each ſmall one will be 1; and if the large diviſions be 100, then each ſmall one will be 10, *&c.*

THEREFORE to lay off a line 8 $\frac{7}{10}$, 87, or 870 parts, let them be leagues, miles, chains, *&c.* ſet one point of the compaſſes on the 8th of the large diviſions, counting from the left hand towards the right, and open the compaſſes, till the other point falls on the 7th of the ſmall diviſions, counting from the right hand towards the left, then are the compaſſes opened to expreſs a line of 8 $\frac{7}{10}$, 87 or 870 leagues, miles, chains, *&c.* and bears ſuch proportion in the plan, as the line meaſured does to the thing repreſented.

But if a length of feet and inches was to be expreſſed, the ſame large diviſions may repreſent the feet, but the inches muſt be taken from the upper part of the firſt diviſion, which (as before noted) is divided into 12 equal parts.

Thus, if a line of 7 feet 5 inches was to be laid down; ſet one point of the compaſſes on the 5th diviſion among the 12, counting from the right hand towards the left, and extend the other to 7, among the large diviſions, and that diſtance laid down in the plan, ſhall expreſs a line of 7 feet 5 inches: and the like is to be underſtood of any other dimenſions.

II. *Diagonally divided.* Draw eleven lines parallel to each other, and at equal diſtances; divide the upper of theſe lines into ſuch a number of equal parts, as the ſcale to be expreſſed is intended to contain, and from each of theſe diviſions draw perpendiculars through the eleven parallels, (Fig. 15.) ſubdivide the firſt of theſe diviſions into 10 equal parts, both in the upper and lower lines; then each of theſe ſubdiviſions may be alſo ſubdivided into 10 equal parts, by drawing diagonal lines; *viz.* from the 10th below, to the 9th above; from the 9th below, to the 8th above; from the 8th below, to the 7th above, *&c.* till from the 1ſt below to the 0th above, ſo that by theſe means one of the primary diviſions on the ſcale, will be divided into 100 equal parts.

There are generally two diagonal ſcales laid on the ſame plane or face of the ruler, one being commonly half the other. (Fig. 15.)

The uſe of the diagonal ſcale is much the ſame with the ſimple ſcale; all the difference is, that a plan may be laid down more accurately by it: becauſe in this, a line may be taken of three denominations; whereas from the former, only two could be taken.

Now from this conſtruction it is plain, if each of the primary diviſions repreſent 1, each of the firſt ſubdiviſions will expreſs $\frac{1}{10}$ of 1; and each of the ſecond ſubdiviſions, (which are taken on the diagonal lines, counting from the top downwards) will expreſs $\frac{1}{10}$ of the former ſubdiviſions, or a 100th of the primary diviſions; and if each of the primary diviſions expreſs 10, then each of the firſt ſubdiviſions will expreſs 1, and each of the 2d, $\frac{1}{10}$; and if each of the primary diviſions repreſent 100, then each of the firſt ſubdiviſions will be 10; and each of the 2d will be 1, *&c.*

Therefore to lay down a line, whoſe length is expreſs'd by 347, $34\frac{7}{10}$ or $3\frac{47}{100}$ whether leagues, miles, chains, *&c.*

On the diagonal line, joined to the 4th of the firſt ſubdiviſions, count 7 downwards, reckoning the diſtance of each parallel 1; there ſet one point of the compaſſes, and extend the other, till it falls on the interſection of the third primary diviſion with the ſame parallel in which the other foot reſts, and the compaſſes will then be opened to expreſs a line of 347, $34\frac{7}{10}$; or $3\frac{47}{100}$, *&c.*

Those who have frequent occaſion to uſe ſcales, perhaps will find, that a ruler with the 20 following ſcales on it, *viz.* 10 on each face, will ſuit more purpoſes than any ſet of ſimply divided ſcales hitherto made public, on one ruler.

| | | |
|---|---|---|
| One Side | The diviſions to an inch | 10, 11, 12, 13½, 15, 16½, 18, 20, 22, 25. |
| Other Side | | 28, 32, 36, 40, 45, 50, 60, 70, 85, 100. |

The left hand primary diviſion, to be divided into 10 and 12 and 8 parts; for theſe ſubdiviſions are of great uſe in drawing the parts of a fortreſs, and of a piece of cannon.

It will here be convenient to ſhew, how any plan expreſſed by right lines and angles, may be delineated by the ſcales of equal parts, and the protractor.

## PROBLEM VIII.

*Three adjacent things in any right lined triangle being given, to form the plan thereof.*

Exam. *Suppoſe a triangular field,* ABC, (Fig. 16.) the ſide AB=327 yards; AC=208 yards; and the angle at A=44½ degrees.

Construction. Draw a line AB at pleaſure; then from the diagonal ſcale take 327 between the points of the compaſſes, and lay it from A to B; ſet the center of the protractor to the point A, lay off 44½ degrees, and by that mark draw AC: take with the compaſſes from the ſame ſcale 208, lay it from A to C, and join CB; ſo ſhall the parts of the triangle ABC, in the plan, bear the ſame proportion to each other, as the real parts in the field do.

The ſide CB may be meaſured on the ſame ſcale from which the ſides AB, AC, were taken: and the angles at B and C may be meaſured by applying the protractor to them as ſhewn at problem IV.

*If two angles and the ſide contained between them were given.*

Draw a line to expreſs the ſide; (as before) at the ends of that line, point off the angles, as obſerved in the field; lines drawn from the ends of the given line through thoſe marks, ſhall form a triangle ſimilar to that of the field.

## PROBLEM IX.

*Five adjacent things, ſides and angles, in a right lin'd quadrilateral, being given, to lay down the plan thereof,* Fig. 17.

EXAM. *Given* $\angle$ * A $= 70^\circ$; AB $= 215$ links; $\angle$ B $= 115^\circ$; BC $= 596$ links; $\angle$ C $= 114^\circ$.

CONSTRUCTION. Draw AD at pleaſure; from A draw AB, ſo as to make with AD an angle of 70°: make AB $= 215$ (taken from the ſcales); from B, draw BC, to make with AB an angle of 115°: make BC $= 596$; from C, draw CD, to make with CB an angle of 114°, and by the interſection of CD with AD, a quadrilateral will be form'd ſimilar to the figure in which ſuch meaſures could be taken as are expreſſed in the example.

IF 3 of the things were ſides, the plan might be formed with equal eaſe.

FOLLOWING the ſame method, a figure of many more ſides may be delineated; and in this manner, or ſome other like to it, do ſome ſurveyors make their plans of ſurveys.

## *The Conſtruction of the remaining Lines of the* PLAIN SCALE.

### PREPARATION. Fig. 18. Pl. VI.

DESCRIBE a circumference with any convenient radius, and draw the diameters AB, DE, at right angles to each other; continue BA at pleaſure towards F; through D, draw DG parallel to BF; and draw the chords BD, BE, AD, AE. Circumſcribe the circle with the ſquare HMN, whoſe ſides HM, MN, ſhall be parallel to AB, ED.

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* This mark or character $\angle$, ſignifies *the angle.*

This mark $=$ ſignifies *equal to.*

By links, is meant the $\frac{1}{100}$th part of a chain of four poles or of 66 yards long.

## I. *To conſtruct the Line of Chords* *.

DIVIDE the arc AD into 90 equal parts; mark the 10th diviſions with the figures 10, 20, 30, 40, 50, 60, 70, 80, 90; on D, as a center, with the compaſſes, transfer the ſeveral diviſions of the quadrantal arc, to the chord AD, which marked with the figures correſponding, will become a line of chords.

*Note*, In the conſtruction of this, and the following ſcales, only the primary diviſions are drawn; the intermediate ones are omitted, that the figure may not appear too much crouded.

* The chord of an arc, is a right line drawn from one end of the arc to the other end.

## II. *The Line of Rhumbs* †.

DIVIDE the arc BE into 8 equal parts, which mark with the figures 1, 2, 3, 4, 5, 6, 7, 8; and divide each of thoſe parts into quarters; on B, as a center, transfer the diviſions of the arc to the chord BE, which marked with the correſponding figures, will be a line of rhumbs.

† The rhumbs here, are the chords anſwering to the points of the mariners compaſs, which are 32 in the whole circle, or 8 in the quarter circle.

## III. *The Line of Sines* ‡.

THROUGH each of the diviſions of the arc AD, draw right lines parallel to the radius AC; and CD will be divided into a line of ſines which are to be numbered

‡ The *ſine of an arc*, is a right line drawn from one end of an arc perpendicular to a radius drawn to the other end.

And the *verſed ſine*, is the part of the radius lying between the arc and its right ſine.

from C to D for the right ſines; and from D to C for the verſed ſines. The verſed ſines may be continued to 180 degrees by laying the diviſions of the radius CD, from C to E.

## IV. The Line of Tangents *.

A RULER on C, and the ſeveral diviſions of the arc AD, will interſect the line DG, which will become a line of tangents, and is to be figured from D to G with 10, 20, 30, 40, &c.

* The *tangent of an arc*, is a right line touching that arc at one end, and terminated by a ſecant drawn through the other end.

## V. The Line of Secants †.

THE diſtances from the center C to the diviſions on the line of tangents being transferred to the line AF from the centre C, will give the diviſions of the line of ſecants; which muſt be numbered from A towards F, with 10, 20, 30, &c.

† The *ſecant of an arc*, is a right line drawn from the centre through one end of an arc, and limited by the tangent of that arc.

## VI. The Line of Half-Tangents (or the Tangents of half the Arcs).

A RULER on E, and the ſeveral diviſions of the arc AD, will interſect the radius CA, in the diviſions of the ſemi, or half tangents; mark theſe with the correſponding figures of the arc AD.

THE ſemi-tangents on the plane ſcales are generally continued as far as the length of the ruler they are laid on will admit; the diviſions beyond 90° are found by dividing the arc AE like the arc AD, then laying a ruler by E and theſe diviſions of the arc AE, the diviſions of the ſemi-tangents above 90 degrees will be obtained on the line CA continued.

## VII. *The Line of Longitude.*

DIVIDE AH, into 60 equal parts; through each of theſe diviſions, parallels to the radius AC, will interſect the arc AE, in as many points; from E as a centre, the diviſions of the arc EA, being transferred to the chord EA, will give the diviſions of the line of longitude.

## VIII. *The Line of Latitude.*

A RULER on A, and the ſeveral diviſions of the ſines on CD, will interſect the arc BD, in as many points; on B as a centre, transfer the interſections of the arc BD, to the right line BD; number the diviſions from B to D, with 10, 20, 30, *&c.* to 90; and BD will be a line of latitude.

## IX. *The Line of Hours.*

BISECT the quadrantal arcs BD, BE, in *a*, *b*; divide the quadrantal arc *ab* into 6 equal parts, (which gives 15 degrees for each hour) and each of theſe into 4 others; (which will give the quarters.) A ruler on C, and the ſeveral diviſions of the arc *ab*, will interſect the line MN in the hour, *&c.* points, which are to be marked as in the figure.

## X. *The Line of Inclinations of Meridians.*

BISECT the arc EA into *c*; divide the quadrantal arc *bc* into 90 equal parts; lay a ruler on C and the ſeveral diviſions of the arc *bc*, and the interſections of the line HM will be the diviſions of a line of inclinations of meridians.

SECT.

## SECT. VIII.

## The uſes of ſome of the Lines on the Plain Scale.

### I. Of the Line of Chords. Pl. VI.

ONE of the uſes of the line of chords is *to lay down a propoſed angle, or to meaſure an angle already laid down.* Thus, to draw a line AC, that ſhall make with the line AB an angle containing a given number of degrees. (ſuppoſe 36.) Figure 19.

ON A, as a centre, with a radius equal to the chord of 60 degrees, deſcribe the arc BC; on this arc, lay the chord of the given number of degrees from the interſection B, to C; draw AC, and the angle BAC will contain the given number of degrees.

*Note,* Degrees taken from the chords are always to be counted from the beginning of the ſcale.

*The degrees contained in an angle already laid down, may be meaſured thus.* Fig. 19.

ON A as a centre, deſcribe an arc BC with the chord of 60 degrees; the diſtance BC, meaſured on the chords, will give the number of degrees contained in the angle BAC.

IF the number of degrees are more than 90; they muſt be taken from, or meaſured by the chords, at twice; thus if 140 degrees were to be protracted, 70° may be taken from the chords, and thoſe degrees laid off twice upon the arc deſcribed with a chord of 60 degrees.

*Note,* Degrees are generally denoted by a ſmall ° put over them.

### II. Of the Line of Rhumbs.

THEIR uſe is to delineate or meaſure a ſhip's courſe; which is the angle made by a ſhip's way and the meridian.

Now having the points and $\frac{1}{4}$ points of the compaſs contained in any courſe; draw a line AB (fig. 19.) for the meridian; on A as a centre, with a chord of 60° deſcribe an arc BC; take the number of points and $\frac{1}{4}$ points from the ſcale of rhumbs, counting from 0, and lay this diſtance on the arc BC, from the interſection B to C; draw AC, and the angle BAC ſhall repreſent the ſhip's courſe.

## III. *The uſe of the Line of Longitude.*

If any two meridians be diſtant one degree or 60 geographical miles, under the equator, their diſtance will be leſs than 60 miles in any latitude between the equator and the pole.

Now let the line of longitude be put on the ſcale cloſe to the line of chords, but inverted; that is, let 60° in the ſcale of longitude be againſt 0° in the chords, and 0° degrees longitude againſt 90° chords. Then mark any degree of latitude counted on the chords; and oppoſite thereto, on the line of longitude, will be the miles contain'd in one degree of longitude, in that latitude.

Thus 57,95 miles, make 1 degree of longitude in the latitude of 15 degrees; 45,97 miles, in latitude 40 degrees; 36,94 miles, in latitude 52 degrees; 30 miles, in latitude 60 degrees, *&c.*

But as the fractional parts are not very obvious on ſcales, here follows a table ſhewing the miles in one degree of longitude to every degree of latitude.

This table is computed, upon the ſuppoſition of the earth being ſpherical, by the following proportion.

As the radius is to the coſine of any latitude, ſo is the miles of longitude under the equator to the miles of longitude in that latitude.

Every perſon who is deſirous of acquiring mathematical knowledge, ſhould have a table of the logarithms of numbers, ſines, tangents, and ſecants; moſt of the treatiſes of navigation, and ſome other books, have theſe tables; but the moſt uſeful and eſteemed are *Sherwin*'s mathematical tables.

## A TABLE, *ſhewing the Miles in one Degree of Longitude to every Degree of Latitude.*

| D. L. | Miles. | D. L. | Miles. | D. L. | Miles. |
|---|---|---|---|---|---|
| 1 | 59,99 | 31 | 51,43 | 61 | 29,09 |
| 2 | 59,96 | 32 | 50,88 | 62 | 28,17 |
| 3 | 59,92 | 33 | 50,32 | 63 | 27,24 |
| 4 | 59,85 | 34 | 49,74 | 64 | 26,30 |
| 5 | 59,77 | 35 | 49,15 | 65 | 25,36 |
| 6 | 59,67 | 36 | 48,54 | 66 | 24,41 |
| 7 | 59,56 | 37 | 47,92 | 67 | 23,44 |
| 8 | 59,42 | 38 | 47,28 | 68 | 22,48 |
| 9 | 59,26 | 39 | 46,63 | 69 | 21,50 |
| 10 | 59,09 | 40 | 45,97 | 70 | 20,52 |
| 11 | 58,89 | 41 | 45,28 | 71 | 19,53 |
| 12 | 58,69 | 42 | 44,59 | 72 | 18,54 |
| 13 | 58,46 | 43 | 43,88 | 73 | 17,54 |
| 14 | 58,22 | 44 | 43,16 | 74 | 16,54 |
| 15 | 57,95 | 45 | 42,43 | 75 | 15,53 |
| 16 | 57,67 | 46 | 41,68 | 76 | 14,52 |
| 17 | 57,38 | 47 | 40,92 | 77 | 13,50 |
| 18 | 57,06 | 48 | 40,15 | 78 | 12,48 |
| 19 | 56,73 | 49 | 39,36 | 79 | 11,45 |
| 20 | 56,38 | 50 | 38,57 | 80 | 10,42 |
| 21 | 56,02 | 51 | 37,76 | 81 | 9,38 |
| 22 | 55,63 | 52 | 36,94 | 82 | 8,35 |
| 23 | 55,23 | 53 | 36,11 | 83 | 7,32 |
| 24 | 54,81 | 54 | 35,27 | 84 | 6,28 |
| 25 | 54,38 | 55 | 34,41 | 85 | 5,23 |
| 26 | 53,93 | 56 | 33,55 | 86 | 4,18 |
| 27 | 53,46 | 57 | 32,68 | 87 | 3,14 |
| 28 | 52,96 | 58 | 31,79 | 88 | 2,09 |
| 29 | 52,47 | 59 | 30,90 | 89 | 1,05 |
| 30 | 51,96 | 60 | 30,00 | 90 | 0,00 |

THE uſes of the ſcales of ſines, tangents, ſecants, and half tangents, are to find the poles and centers of the ſeveral circles repreſented in the orthographical and ſtereographical projection of the ſphere; which are reſerved until the explanation and uſe of the lines of the ſame name on the ſector are ſhewn.

THE lines of latitudes, hours, and inclinations of meridians, are applicable to the practice of dialing; on which there are ſeveral treatiſes extant, which may be conſulted.

## SECT. IX.

## *Of the* SECTOR.

A Sector is a figure form'd by two radius's of a circle, and that part of the circumference comprehended between the two radius's.

THE inſtrument called a ſector, conſiſts of two rulers moveable round an axis or joint, from whence ſeveral ſcales are drawn on the faces of the rulers.

THE two rulers are called legs, and repreſent the radii, and the middle of the joint expreſſes the center.

THE ſcales generally put on ſectors, may be diſtinguiſhed into ſingle, and double.

THE ſingle ſcales are ſuch as are commonly put on plain ſcales, and from whence dimenſions or diſtances are taken as have been already directed.

THE double ſcales are thoſe which proceed from the center; each ſcale is laid twice on the ſame face of the inſtrument, *viz.* once on each leg: From theſe ſcales, dimenſions or diſtances are to be taken, when the legs of the inſtrument are in an angular poſition, as will be ſhewn hereafter.

*The Scales commonly put on the beſt Sectors, are*

| | | | | | |
|---|---|---|---|---|---|
| Single | 1 | a line of | Inches, each Inch divided into 8 and 10 parts. | | |
| | 2 | | Decimals, containing an 100 parts. | | |
| | 3 | | Chords, | mark'd | Cho. |
| | 4 | | Sines, | | Sin. |
| | 5 | | Tangents, | | Tang. |
| | 6 | | Rhumbs, | | Rum. |
| | 7 | | Latitude, | | Lat. |
| | 8 | | Hours, | | Hou. |
| | 9 | | Longitude, | | Lon. |
| | 10 | | Inclin. Merid. | | In.Me |
| | 11 | | the Logarithms of Numbers, | | Num. |
| | 12 | | the Logarithms of Sines, | | Sin. |
| | 13 | | the Logarithms of Verſed Sines, | | V.Sin. |
| | 14 | | the Logarithms of Tangents, | | Tan. |

| | | | | | |
|---|---|---|---|---|---|
| Double | 1 | a line of | Lines, or of equal parts, | mark'd | Lin. |
| | 2 | | Chords. | | Cho. |
| | 3 | | Sines. | | Sin. |
| | 4 | | Tangents to 45° | | Tan. |
| | 5 | | Secants, | | Sec. |
| | 6 | | Tangents to above 45° | | Tan. |
| | 7 | | Polygons, | | Pol. |

The manner in which theſe ſcales are diſpoſed of on the ſector, is beſt ſeen in the plate fronting the title page.

The ſcales of lines, chords, ſines, tangents, rhumbs, latitudes, hours, longitude, incl. merid. may be uſed, whether the inſtrument is ſhut or open, each of theſe ſcales being contained on one of the legs only. The ſcales of inches, decimals, log. numbers, log. ſines, log. verſed ſines and log. tangents, are to be uſed with the ſector quite opened, part of each ſcale lying on both legs.

The double ſcales of lines, chords, ſines, and lower tangents, or tangents under 45 degrees, are all of the ſame radius or length; they begin at the center of the inſtrument, and are terminated near the other extremity of each leg; *viz.* the lines at the diviſion 10, the chords at 60, the ſines at 90, and the tangents at 45; the remainder of the tangents, or thoſe above 45°, are on other ſcales beginning at $\frac{1}{4}$ of the length of the former, counted from the center, where they are marked with 45, and run to about 76 degrees.

THE ſecants alſo begin at the ſame diſtance from the center, where they are marked with 10, and are from thence continued to as many degrees as the length of the ſector will allow, which is about 75°.

THE angles made by the double ſcales of lines, of chords, of ſines, and of tangents to 45 degrees, are always equal.

AND the angles made by the ſcales of upper tangents, and of ſecants, are alſo equal; and ſometimes theſe angles are made equal to thoſe made by the other double ſcales.

THE ſcales of polygons are put near the inner edge of the legs, their beginning is not ſo far removed from the center, as the 60 on the chords is: Where theſe ſcales begin, they are mark'd with 4, and from thence are figured backwards, or towards the center, to 12.

FROM this diſpoſition of the double ſcales, it is plain, that thoſe angles which were equal to each other, while the legs of the ſector were cloſe, will ſtill continue to be equal, although the ſector be opened to any diſtance it will admit of.

## SECT. X.

## *Of the Conſtruction of the Single Scales.*

### I. *The Scale of Inches.*

THIS ſcale, which is laid cloſe to the edge of the ſector, and ſometimes on the edge, contains as many inches as the inſtrument will receive when opened: Each inch is uſually divided into 8 equal parts, and alſo into 10 equal parts.

### II. *The Decimal Scale.*

THIS ſcale lies next to the ſcale of inches; it is of the ſame length of the ſector when opened, and is divided into 10 equal parts, or primary diviſions; and each of theſe into 10 other equal parts; ſo that the whole is divided into 100 equal parts. And where the ſector is long enough, each of the ſubdiviſions is divided into two, four, or five parts; and by this decimal ſcale, all the other ſcales, that are taken from tables, may be laid down.

THE length of a ſector is uſually underſtood when it is ſhut, or the legs cloſed together. Thus a ſector of ſix inches when ſhut, makes a ruler of twelve inches when opened, and a foot ſector, is two feet long when quite opened.

### III. *The Scales of Chords, Rhumbs, Sines, Tangents. Hours, Latitudes, Longitudes, and Inclination of Meridians;*

ARE ſuch as have been already deſcribed in the account of the plane ſcale.

## IV. *The Scale of Logarithmic Numbers.*

THIS ſcale, commonly called the artificial numbers, and by ſome the *Gunter*'s ſcale, or *Gunter*'s * line, is a ſcale expreſſing the logarithms of common numbers, taken in their natural order. To lay down the diviſions in the beſt manner, there is neceſſary a good table of logarithms, (ſuppoſe *Sherwin*'s,) and a ſcale of equal parts, accurately divided, and of ſuch a length, that 20 of the primary diviſions ſhall make the whole length of the intended ſcale of numbers, or logarithm ſcale.

### *The Conſtruction.*

1. FROM the ſcale of equal parts, take the firſt 10 of the primary diviſions, and lay this diſtance down twice on the log. ſcale, making two equal intervals; marking the firſt point 1, the ſecond 1, (or rather 10) and the third 10, (or rather 100.)

2. FROM the ſcale of equal parts, take the diſtances expreſſed by the logs. of the numbers, 2, 3, 4, 5, 6, 7, 8, 9, reſpectively, (rejecting the indices :) lay theſe diſtances on each interval of the log. ſcale, between the marks 1 & 10, 10 & 100, reckoning each diſtance from the beginning of its interval, *viz.* from 1, and from 10, and mark theſe diſtances with the figures 2, 3, 4, 5, 6, 7, 8, 9, in order.

THUS the firſt three figures of the logarithms of 2, 3, 4, 5, 6, 7, 8, 9, are, 301, 477, 602, 699, 778, 845, 903, 954; theſe are the numbers that are to be taken from the ſcale of equal parts, and laid down in each interval, obſerving that the extent for each is to be applied from the beginning of the intervals.

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* From Mr. *Edmund Gunter*, the Inventor: Aſtronomy-Profeſſor in *Greſham-College*, Anno 1624.

3. The diſtances expreſſing the logs. of the numbers between 10 & 20, 20 & 30, 30 & 40, 40 & 50, 50 & 60, 60 & 70, 70 & 80, 80 & 90, 90 & 100, (rejecting the indices) are alſo to be taken from the ſcale of equal parts, and laid on the log. ſcale, in each of the primary intervals, between the marks 1 & 2, 2 & 3, 3 & 4, 4 & 5, 5 & 6, 6 & 7, 7 & 8, 8 & 9, 9 & 10, reſpectively; reckoning each diſtance from the beginning of its reſpective primary interval.

4. The laſt ſubdiviſions of the ſecond primary interval are to be divided into others, as many as the ſcale will admit of, which is done by laying down the logarithms of ſuch intermediate diviſions, as it ſhall be thought proper to introduce.

## V. *The Scale of Logarithm Sines.*

1. From the ſcale of equal parts, take the diſtances expreſſed by the arithmetical complements * of the logarithmic ſines, (or the ſecants of the complements) of 80, 70, 60, 50, 40, 30, 20, 10, degrees reſpectively; rejecting the indices; and theſe diſtances, lay on the ſcale of log. ſines, reckoning each from the mark intended to expreſs 90 degrees.

Thus. To the ſines of 80°, 70°, 60°, 50°, 40°, 30°, 20°, 10°, the three firſt figures of the arithmetical complements of their logarithms, are, 007, 026, 063, 115, 192, 301, 466, 760; theſe are the numbers to be taken from the ſcale of equal parts, uſed for laying down the logarithms of numbers, and every extent of the compaſſes is to be laid from the right hand towards the left, beginning at the point choſe for 90°, which uſually ſtands directly under the end of the line of numbers.

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* By the arithmetical complement of any ſine, tangent, *&c.* is meant the remainder, when that ſine, tangent, *&c.* is ſubſtracted from radius, or 10,000000, *&c.*

2. In the ſame manner, lay off the degrees under 10; and alſo, the degrees intermediate to thoſe of 10, 20, 30, &c.

3. Lay down as many of the multiples of 5 minutes, as may conveniently fall within the limits of thoſe degrees which will admit of ſuch ſubdiviſions of minutes.

## VI. *The Scale of Logarithmic Tangents.*

1. This ſcale, as far as 45 degrees, is conſtructed, in every particular, like that of the log. ſines; uſing the arithmetical complements of the log. tangents.

2. The degrees above 45, are to be counted backwards on the ſcale: Thus 40 on the ſcale, repreſents both 40 degrees, and 50 degrees; 30 on the ſcale, repreſents both 30 degrees, and 60 degrees; and the like of the other mark'd degrees, and alſo of their intermediate ones.

## VII. *The Logarithmic verſed Sines.*

1. From the ſcale of equal parts, take the arithmetical complements of the logarithm co-ſines, (or the ſecants of the complements) of 5, 10, 15, 20, 25, 30, 35, 40, &c. degrees; (rejecting the indices,) and the double of theſe diſtances, reſpectively, laid on the ſcale (intended) for the log. verſed ſines, will give the diviſions expreſſing 10, 20, 30, 40, 50, 60, 70, 80, &c. degrees; to as many as the length of the ſcale will take in.

2. Between every diſtance of 10 degrees, introduce as many degrees, $\frac{1}{2}$ degrees; $\frac{1}{3}$ degrees; $\frac{1}{4}$ degrees, &c. as the intervals will admit.

The ſcales of the logarithms of numbers, ſines, verſed ſines, and tangents, ſhould have one common termination to one end of each ſcale; that is, the 10 on the numbers, the 90 on the ſines, the 0 on the verſed ſines, and the 45 on the tangents, ſhould be oppoſite to each other: The other end of each of the ſcales of ſines, verſed ſines, and tangents, will run out beyond the beginning (mark'd 1) of the numbers; nearly oppoſite to which, will be the diviſions repreſenting 35 minutes on the ſines and tangents, and $168\frac{1}{2}$ degrees, on the verſed ſines.

## Sect. XI.

## *Of the Conſtruction of the Double Scales.*

### I. *Of the Line of Lines.*

THIS is only a ſcale of equal parts, whoſe length is adapted to that of the legs of the ſector: Thus in the ſix inch ſector, the length is about $5\frac{3}{4}$ inches.

The length of this ſcale is divided into 10 primary diviſions; each of theſe into 10 equal ſecondary parts; and each ſecondary diviſion, into 4 equal parts.

Hence on any ſector it will be eaſy to try if this line is accurately divided: Thus. Take between the compaſſes any number of equal parts from this line, and apply that diſtance to all the parts of the line; and if the ſame number of diviſions are contained between the points of the compaſſes in every application, the ſcale may be received as perfect.

## II. *Of the Line of Sines.*

1. MAKE the whole length of this ſcale, equal to that of the line of lines.

2. FROM the ſcale of the line of lines, take off ſeverally, the parts expreſſed by the numbers in the tables (ſuppoſe *Sherwin*'s) of the natural ſines, correſponding to the degrees, or to the degrees and minutes, intended to be laid on the ſcale.

3. LAY down theſe diſtances ſeverally on the ſcale, beginning from the center; and this will expreſs a ſcale of natural ſines.

EXAM. *To lay down* 35° 15′; *whoſe natural ſine found in the tables is* 57714, *&c.*

TAKE this number as accurately as may be, from the line of lines, counting from the center; and this diſtance will reach from the beginning of the ſines, at the center of the inſtrument, to the diviſion expreſſing 35° 15′; and ſo of the reſt.

IN ſcales of this length, it is cuſtomary to lay down diviſions, expreſſing every 15 minutes, from 0 degrees to 60 degrees; between 60 and 80 degrees, every half degree is expreſſed; then every degree to 85; and the next, is 90 degrees.

## III. *Of the Scale of Tangents.*

THE length of this ſcale is equal to that of the line of lines, and the ſeveral diviſions thereon (to 45 degrees) are laid down from the tables and line of lines, in the ſame manner as has been deſcribed in the ſines; obſerving to uſe the natural tangents in the tables.

## IV. *Of the Scale of upper Tangents.*

THIS ſcale is to be laid down, by taking $\frac{1}{4}$ of ſuch of the natural tabular tangents above 45 degrees, as are intended to be put on the ſcale.

ALTHOUGH the poſition of this ſcale on the ſector reſpects the center of the inſtrument, yet its beginning, at 45 degrees, is diſtant from the center, $\frac{1}{4}$ of the length or radius of the lower tangents.

## V. *Of the Scale of Secants.*

THE diſtance of the beginning of this ſcale, from the center, and the manner of laying it down, is juſt the ſame as that of the upper tangents; only in this, the tabular ſecants are to be uſed.

## VI. *Of the Scale of Chords.*

1. MAKE the length of this ſcale, equal to that of the ſines; and let the diviſions to be laid down, expreſs every 15 minutes from 0 degrees to 60 degrees.

2. TAKE the length of the ſine of half the degrees and minutes, for every diviſion to be laid down, (as before directed in the ſcale of ſines;) and twice this length, counted from the center, will give the diviſions required.

THUS, twice the length of the ſine 18° 15′, will give the chord of 36° 30′; and in the ſame manner for the reſt.

## VII. *Of the Scale of Polygons.*

THIS ſcale uſually takes in the ſides of the polygons from 6 to 12 ſides incluſive: The diviſions are laid down, by taking the lengths of the chords of the angles at the center of each polygon; and theſe diſtances are laid from the center of the inſtrument.

But it is beſt to have the polygons of 4 and 5 ſides alſo introduced; and then this line is conſtructed from a ſcale of chords, where the length of 90 degrees is equal to that of 60 degrees of the double ſcale of chords on the ſector.

In the place of ſome of the double ſcales here deſcribed, there are found other ſcales on the old ſectors, and alſo on ſome of the modern *French* ones, ſuch as, ſcales of ſuperficies, of ſolids, of inſcribed bodies, of metals, *&c.* But theſe ſeem to be juſtly left out on the ſectors, as now conſtructed, to make room for others of more general uſe: However, theſe ſcales, and ſome others, of uſe in gunnery, ſhall hereafter be deſcribed in a tract on the uſe of the gunners callipers.

## SECT. XII.

### *Of the Uſes of the Double Scale.*

IN the following account of the uſes, as there will frequently occur the terms *lateral diſtance*, and *tranſverſe diſtance*; it will be proper to explain what is meant by thoſe terms.

*Lateral diſtance*, is a diſtance taken by the compaſſes on one of the ſcales only, beginning at the center of the ſector.

*Tranſverſe diſtance*, is the diſtance taken between any two correſponding diviſions of the ſcales of the ſame name, the legs of the ſector being in an angular poſition: That is, one foot of the compaſſes is ſet on a diviſion in a ſcale on one leg of the ſector, and the other foot is extended to the like diviſion in the ſcale of the ſame name on the other leg of the ſector.

It muſt be obſerved, that each of the ſectoral ſcales have three parallel lines, acroſs which the diviſions of the ſcale are marked: Now in taking tranſverſe diſtances, the points of the compaſſes muſt be always ſet on the inſide line, or that line next the inner edge of the leg; for this line only in each ſcale runs to the center.

## *Some Uſes of the Line of Lines.*

### PROBLEM X.

*To two given lines* AB = 2, BC = 6; *to find a third proportional.* Plate VI. Fig. 20.

Operation. 1. Take between the compaſſes, the lateral diſtance of the ſecond term, (*viz.* 6.)

2. Set one point on the diviſion expreſſing the firſt term (*viz.* 2.) on one leg, and open the legs of the ſector till the other point will fall on the correſponding diviſion on the other leg.

3. Keep the legs of the ſector in this poſition; take the tranſverſe diſtance of the ſecond term, (*viz.* 6.) and this diſtance is the third term required.

4. This diſtance meaſured laterally, beginning from the center, will give (18) the number expreſſing the meaſure of the third term: For 2 : 6 :: 6 : 18.

Or, Take the diſtance 2 laterally, and apply it tranſverſely to 6 and 6 (the ſector being properly opened), then the tranſverſe diſtance at 2 and 2 being taken with the compaſſes and applied laterally from the center of the ſector on the ſcale of lines, will give ,66$\frac{2}{3}$ = $\frac{2}{3}$, the third term when the proportion is decreaſing: For 6 : 2 :: 2 : $\frac{2}{3}$.

*Note*, If the legs of the ſector will not open ſo far as to let the lateral diſtance of the ſecond term fall between the diviſions expreſſing the firſt term; then take $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, or any aliquot part of the ſecond term, (ſuch as will conveniently fall within the opening of the ſector) and make ſuch part, the tranſverſe diſtance of the firſt term; then if the tranſverſe diſtance of the ſecond term be multiplied by the denominator of the part taken of the ſecond term, the product will give the third term.

## PROBLEM XI.

*To three given lines* AB = 3, BC = 7, CD = 10; *to find a fourth proportional.* Plate VI. Fig. 21.

Operation. Open the legs of the ſector, until the tranſverſe diſtance of the firſt term, (3) be equal to the lateral diſtance of the ſecond term, (7) or to ſome part thereof; then will the tranſverſe diſtance of the third term, (10) give the fourth term, ($23\frac{1}{3}$) required; or, ſuch a ſubmultiple thereof as was taken of the ſecond term: For 3 : 7 :: 10 : $23\frac{1}{3}$

Or, Set the lateral diſtance 7 tranſverſely from 10 to 10 (opening the ſector properly); then the tranſverſe diſtance at 3 and 3 taken and applied laterally, will give $2\frac{1}{10}$: For 10 : 7 :: 3 : $2\frac{1}{10}$.

From this problem is readily deduced, how to increaſe or diminiſh a given line, in any aſſigned proportion.

Exam. *To diminiſh a line of* 4 *inches, in the proportion of* 8 *to* 7.

1. Open the ſector until the tranſverſe diſtance of 8 and 8, be equal to the lateral diſtance of 7.

2. Mark the point to where 4 inches will reach, as a lateral diſtance taken from the center.

3. The tranſverſe diſtance, taken at that point, will be the line required.

If the given line, ſuppoſe 12 inches, ſhould be too long for the legs of the ſector, take $\frac{1}{2}$, or $\frac{1}{3}$, or $\frac{1}{4}$, &c. part of the given line for the lateral diſtance;

and the correſponding tranſverſe diſtance, taken twice, or thrice, or four times, *&c.* will be the line required.

## PROBLEM XII.

*To open the ſector ſo, that the two ſcales of lines ſhall make a right angle.*

Operation. Take the lateral diſtance from the center to the diviſion marked 5 between the points of the compaſſes, and ſet one foot on the diviſion marked 4 on one of the ſcales of lines, and open the legs of the ſector till the other foot falls on the diviſion marked 3 on the other ſcale of lines, and then will thoſe ſcales ſtand at right angles to one another.

For the lines 3, 4, 5, or any of their multiples, conſtitute a right angle triangle.

## PROBLEM XIII.

*To two right lines given, to find a mean proportional. Suppoſe the lines* 40 *and* 90.

Operation. 1ſt. Set the two ſcales of lines at right angles to one another.

2d. Find the half ſum of the given lines ($= \frac{90+40}{2} = 65$); alſo find the half difference of thoſe lines ($= \frac{90-40}{2} = 25$).

3d. Take, with the compaſſes, the lateral diſtance of the half ſum (65), and apply one foot to the half difference (25), the other foot tranſverſely will reach to (60) the mean proportional required: For 40 : 60 :: 60 : 90.

PROBLEM

## PROBLEM XIV.

*To divide a given line into any propoſed number of equal parts: (ſuppoſe* 9).

Make the length of the given line, or ſome known part thereof, a tranſverſe diſtance to 9 and 9: Then will the tranſverſe diſtance of 1 and 1, be the $\frac{1}{9}$ part thereof; or ſuch a ſubmultiple of the $\frac{1}{9}$ part, as was taken of the given line.

Or the $\frac{1}{9}$ part, will be the difference between the given line, and the tranſverſe diſtance of 8 and 8.

The latter of theſe methods is to be preferred when the part required falls near the center of the inſtrument.

*To this problem may be referred the method of making a ſcale of a given length, to contain a given number of equal parts.*

The practice of this is very uſeful to thoſe who have occaſion to take copies of ſurveys of lands; draughts of buildings, whether civil or military; and in every other caſe, where drawings are to be made to bear a given proportion to the things they repreſent.

Exam. *Suppoſe the ſcale to the map of a ſurvey is* 6 *inches long, and contains* 140 *poles; required to open the ſector ſo, that a correſponding ſcale may be taken from the line of lines.*

Solution. Make the tranſverſe diſtance 7 and 7 (or 70 and 70, *viz.* $\frac{140}{2}$) equal to three inches ($= \frac{6}{2}$); and this poſition of the line of lines will produce the given ſcale.

*If it was required to make a ſcale of* 140 *poles, and to be only two inches long.*

Solution. Make the tranſverſe diſtance of 7 and 7 equal to one inch, and the ſcale is made.

Exam. II. *To make a ſcale of* 7 *inches long contain* 180 *fathoms.*

Solution. Make the tranſverſe diſtance of 9 and 9 equal to $3\frac{1}{2}$ inches, and the ſcale is made.

Exam. III. *To make a ſcale which ſhall expreſs* 286 *yards, and be* 18 *inches long.*

Solution. Make the $\frac{1}{3}$ of 18 inches (or 6 inches) a tranſverſe diſtance to the $\frac{1}{3}$ of 286 ($= 95\frac{1}{3}$) and the ſcale is made.

Or, Make the $\frac{1}{4}$ of 18 inches ($= 4\frac{1}{2}$ inches) a tranſverſe diſtance to $\frac{1}{4}$ of 286 ($= 71\frac{1}{2}$), and the ſcale is made.

Exam. IV. *To divide a given line (ſuppoſe of* 5 *inches) into any aſſigned proportion (as of* 4 *to* 5*).*

Solution. Take (5 inches) the length of the given line, between the compaſſes, and make this a tranſverſe diſtance to (9 and 9) the ſum of the propoſed parts; then the tranſverſe diſtances of the aſſigned numbers (4 and 5) will be the parts required.

## PROBLEM XV.

*The uſe of the line of lines in drawing the orders of Civil Architecture.*

In this place it is intended to give ſo much of Architecture as may enable a beginner to draw any one of the orders; but that the following precepts may be rightly underſtood, it will be proper to explain a few of the terms.

### Definitions.

1. Architecture is the art of building well; and has for its object the Convenience, Strength, and Beauty of the building.

2. ORDER in Architecture, is generally underſtood as Ornament, and conſiſts of three grand parts, namely;

3. THE ENTABLATURE, which repreſents, or is, the weight to be ſupported.

4. THE COLUMN, that which ſupports any weight.

5. THE PEDESTAL or foot whereon the Column is ſet for its better ſecurity.

EACH of theſe parts conſiſts alſo of three parts.

6. THE *Pedeſtal* is compoſed of a BASE, or lower part, a DIE, and a CORNICE, or upper part.

7. THE *Column* is made up of a BASE, a SHAFT, which is a middle part, and a CAPITAL, the upper part.

8. THE *Entablature* conſiſts of an ARCHITRAVE, or lower part, a FREEZE, the middle part, and a CORNICE, the upper part.

So that an Order may be ſaid to conſiſt of nine large parts, each of which is made up of ſmaller parts called Members; whereof ſome are *Plane*, ſome *Curved*, either convex or concave, or convexo-concave.

PLANE members of different magnitude have different names.

9. A FILLET or *liſt* is the leaſt plane or flat member.

10. A PLINTH is that flat member at the bottom of the Pedeſtal, or of the baſe of the Column.

11. A PLATEBAND, that at the top of the Pedeſtal, or the upper member of the Architrave in the Entablature.

12. AN ABACUS, that at the top of the capital.

13. THE FACIÆ or faces are flat members in the Architrave.

14. THE CORONA is a large flat member in the Cornice.

THE Convex members are,

15. AN ASTRAGAL of a ſmall ſemicircular convexity.

16. A Fusarole when an Aſtragal is cut into parts like beads.

17. A Torus a large ſemicircular convexity.

18. An Ovola nearly of a quadrantal convexity.

The Concave members are,

19. A Cavetto nearly of a quadrantal concavity.

20. A Scotie of a concavity nearly ſemicircular.

The Convexo-Concave members are a Cymaiſe and a Cima.

21. A Cymaise or *Ove*, that whoſe convex part projects moſt; and by workmen is uſually called an Ogee.

22. A Cima that whoſe concave part projects moſt.

23. Soffit is the under part of the Crown of an Arch, or of the Corona of an Entablature.

24. Trigliphs *(i. e.* three channels) is an Ornament in the Freeze of the Doric Order.

25. Metops *(i. e.* between three's) is the ſpace of the Freeze between two Trigliphs.

26. Modilions, or Mutules, are the brackets or ends of beams ſupporting the Corona. In the Corinthian Order they are generally carved into a kind of Scrol.

27. Dentels are an Ornament looking ſomewhat like a row of teeth; and are placed in the Cornice of the Entablature.

It is cuſtomary among Architects to eſtimate the heights and projections of all the parts of every order by the diameter of the column at the bottom of the ſhaft, which they call a module; and ſuppoſe it to conſiſt of 60 equal parts, which are called minutes.

## *Of the* TUSCAN ORDER.

This order, which ſome writers liken to a ſtrong robuſt labouring man, is the moſt ſimple and unadorned of any of the orders: The places moſt recommended to uſe it in, are country farm-houſes, ſtables, gateways to inns, and places where plainneſs and ſtrength are reckoned moſt neceſſary: Though there are inſtances where this order has been applied to buildings of a more public and elegant nature.

THE general proportions aſſigned by *Palladio*.

1. HEIGHT of the column equal to ſeven diameters, or modules.
2. HEIGHT of the entablature equal to one fourth of the column, wanting half a minute.
3. HEIGHT of the pedeſtal equal to one module.
4. THE capital and baſe, each half a module.
5. BREADTH of the baſe on a level is $1\frac{1}{6}$ module.
6. BREADTH of the capital equal to one module.
7. DIMINISHING of the column is $\frac{1}{4}$ module.
8. PROJECTION of the beams ſupporting the eaves is $1\frac{3}{4}$ modules.
9. IN colonades, the diſtance of the columns in the clear is 4 modules.
10. IN arches, and the columns ſet on pedeſtals,
    - The diſtance of the columns from middle to middle is $6\frac{5}{12}$ modules.
    - Height of the arch is $7\frac{2}{3}$ modules.
    - Breadth of the pilaſter between the column and paſſage is 26 minutes.

THE ovolo under the corona, in the cornice of the entablature, is commonly continued within the corona, giving it a reverſe bending in the ſoffit, ſomething like a cyma.

## *Of the* DORIC ORDER.

THIS order, ſuppoſed to be invented by *Dorus* a king of *Achaia*, may be likened to a well limbed genteel man; and although of a bold aſpect, yet not ſo ſturdy and ruſticly clad as the Tuſcan. Architects place this order indifferently in towns: But when they would decorate a country ſeat with it, the open champaign ſituation ſeems beſt for the reception of the

Doric order; notwithſtanding which, there are many fine buildings of this order in other ſituations, where they have a very pleaſing effect.

THE following general proportions are given by *Palladio*.

1. HEIGHT of the column from $7\frac{1}{2}$ to 8, and $8\frac{2}{3}$ modules.
2. HEIGHT of the entablature is one fourth of the column.
3. HEIGHT of the pedeſtal equal to $2\frac{1}{3}$ modules.
4. THE Attic baſe is uſed with this order, it is half a module in height, and ſo is the capital.
5. BREADTH of the column's baſe is $1\frac{1}{6}$ module.
6. BREADTH of the capital is 1 module $17\frac{1}{2}$ minutes.
7. DIMINISHING of the column is 8 minutes.
8. IN colonades, the diſtance of the columns in the clear is $2\frac{3}{4}$ modules.
9. IN arches, and the column ſet on pedeſtals,
    - Diſtance of the columns from middle to middle is $7\frac{1}{2}$ modules.
    - Height of the arch to its ſoffit is $10\frac{1}{4}$ modules.
    - Breadth of the pilaſters is 26 minutes.

IN the Doric order the architrave has two faces and a plinth; the upper face is ornamented with rows of ſix *drips* or *bells*, covered with a plain cap: The freeze is divided into trigliphs and metops: The breadths of the drips, cap and trigliphs are each $\frac{1}{2}$ module: The trigliphs conſiſt of two channels, two half channels, and three voids; the breadths of the channels and voids are each 5 minutes: The axis of the column continued, runs through the middle void, leaving the drips three on each ſide: The metops, or diſtances between the trigliphs, are equal to the height of the freeze, and are commonly ornamented with trophies, arms, roſes, *&c.*

Here follows a table for the particular conſtruction of the ornaments with which the architrave and freeze are enriched.

| | Altitude. | Projection | Profile. |
|---|---|---|---|
| | Min. | Min. | Min. |
| Capital | 5 | 16 | 3 |
| Freeze | 45 | — | — |
| Trigliphs | 40 | 15 | $\frac{1}{2} + 2\frac{1}{2}$ |
| Plinth | $4\frac{1}{2}$ | 16 | 3 |
| Cap | $1\frac{2}{3}$ | 15 | 2 |
| Drips | $3\frac{1}{3}$ | 15 | 2 |

The column ſigned altitude gives the heights of the particular parts.

That ſigned projection ſhews the breadths of thoſe parts on each ſide of the middle line of the column continued.

And under the word profile ſtand the numbers ſhewing how far the ſeveral parts project beyond the planes or faces of the members on which they are made.

The ſoffit of the corona in the cornice of the entablature, is uſually ornamented with drips correſponding to the trigliphs, and roſes, arms, *&c.* over the metops.

The ſhaft of the column is ſometimes fluted; that is, cut into channels from top to bottom, the channels meeting one another in an edge, and are in number twenty.

## *Of the* IONIC ORDER.

This order, which is taller and ſlenderer than the Doric, does not appear with ſuch a maſculine ſtrength;

and is by ſome writers compared to the figure of a grave matron. The *Ionians* who invented this order, applied it chiefly to decorate their temples: But when applied to the ornamenting a country palace, the rich and extended vale ſeems a proper ſite: Workmen indeed uſe it indifferently in every place.

*Palladio* gives to the Ionic order the following general proportions.

1. HEIGHT of the column to be 9 modules.

2. THE altitude of the entablature is equal to $\frac{1}{5}$ that of the column, and divided for the architrave, freeze, and cornice, in the proportion of 4, 3, 5.

3. THE height of the pedeſtal equal to 2 modules $37\frac{1}{2}$ minutes; or $\frac{8}{27}$ of the column.

4. HEIGHT of the baſe $\frac{1}{2}$ module; its breadth 1 module $22\frac{1}{2}$ minutes.

5. HEIGHT of the capital and volute is $31\frac{2}{3}$ minutes, and the breadth of its abaco is 1 module $3\frac{1}{3}$ minutes.

6. DIMINUTION of the column is $7\frac{1}{4}$ minutes.

7. IN colonades, the diſtance of the columns in the clear is $2\frac{1}{4}$ modules.

8. IN arches, and the columns ſet on pedeſtals,
   Diſtance of the columns from middle to middle is $7\frac{7}{24}$ modules.
   Height of the arch to its ſoffit is 11 modules.
   Breadth of the pilaſters is $26\frac{1}{2}$ minutes, between the column and arch.

THE diſtance of the modilions in the entablature is 22 minutes, and the breadth of each modilion is 10 minutes; the axis of the column produced always paſſes through the middle of a modilion, which in this order is a plain block repreſenting the end of a beam. The three moſt elegant remains of the ancient Ionic order in *Rome* have their cornice ornamented with dentels inſtead of modilions; and it is the opinion of ſome, eminent for their taſte in Architecture, that in this order dentels would have a better effect than modilions; the heights of theſe dentels were uſually twice their breadth, and their diſtances half their breadth.

The freeze of this order is uſually made ſwelling, and is formed by the ſegment of a circle, whoſe chord is parallel to the axis of the column, and the ſwelling projecting as far as the plateband of the architrave.

The volutes of the capital are now made to project in the directions of the diagonals of the ſquare cap, or abaco, over the volutes, ſo that their drawing ſhould be expreſſed like the volutes in the Roman order: They are much better drawn by an eaſy hand, than by any rules for deſcribing them with the compaſſes, obſerving the limits of their altitude and projection: But the volutes in the ancient examples of this order were curled in a plane parallel to the architrave. Theſe volutes are ſuppoſed to repreſent the plaited treſſes in which the Grecian women uſed to dreſs their hair.

The ſhaft of the column is ſometimes fluted, leaving a fillet or liſt between each channel: In this order there are 24 flutes and fillets.

## *Of the* CORINTHIAN ORDER.

This order, the moſt elegant of all, is by ſome compared to a very fine woman clad in a wanton ſumptuous habit: It was invented at *Corinth*, and ſoon ſpread into other places to adorn their public buildings. A proper rural ſituation for this order, ſeems to be a ſpot commanding a rich and beautiful proſpect in a fine watered vale.

The general proportions aſſigned by *Palladio* are;

1. The height of the column to be $9\frac{1}{2}$ modules.

2. Height of the entablature equal to $\frac{1}{5}$ that of the column; the architrave, freeze and cornice to be in the proportion of 4, 3, 5; and the projection of the cornice equal to its height.

3. Height of the pedeſtal equal to $\frac{1}{4}$ of the column.

4. The height of the capital to be $1\frac{1}{6}$ module; of which the abaco is $\frac{1}{6}$ of a module; its horns projecting over the bottom of the column $\frac{1}{4}$ of a module.

5. The height of the baſe equal to $\frac{1}{2}$ module; and its greateſt breadth to be one module and a fifth.

6. The diminution of the column to be 8 minutes.

7. In colonades, the intercolumniation is 2 modules.

8. In arches, and the columns ſet on pedeſtals,

The diſtance of the columns, from middle to middle, to be $6\frac{1}{2}$ modules.

Height of the arch equal to $11\frac{1}{6}$ modules.

Breadth of the pilaſter, between the column and ſides of the paſſage, to be 27 minutes.

In this order, the ſhaft is frequently cut into 24 flutes, which are ſeparated from one another by as many fillets.

The capital is compoſed of three tiers of leaves, eight leaves in a tier, with their ſtalks or ſcrols, encircling the body of the capital, which repreſents a baſket, whoſe bottom is juſt as broad as the diameter of the top of the column within the channels: The ornaments of this capital are beſt done by hand, without rule or compaſs, obſerving the proper altitudes and projections of the parts.

The architrave conſiſts of three faciæs, three fuſaroles, an ogee, and a plateband; the firſt, or lower faciæ projects the ſame as the top of the ſhaft.

The freeze, which projects the ſame as the top of the ſhaft, has its lower part turned into a kind of cavetto, terminating with the extremity of the plateband of the architrave.

The breadths of the dentels are $3\frac{1}{3}$ minutes, and their diſtance $1\frac{2}{3}$ minutes.

The breadths of the modilions are $11\frac{1}{3}$ minutes, and their diſtance in the clear $23\frac{1}{4}$ minutes.

THE middle of a dentel ſhould be under the middle of a modilion, and the axis of the column paſſes through the middles of both dentel and modilion.

## *Of the* COMPOSITE ORDER.

THIS order (the poor invention of the *Romans*, and therefore frequently called the Roman order), is uſually compoſed of the Corinthian and Ionic; the Ionic capital being ſet over the two lower rows of leaves in the Corinthian capital.

*Palladio* gives us the following general proportions.

1. THE height of the column to be 10 modules.

2. THE height of the entablature equal to $\frac{1}{5}$ of the column; the architrave, freeze, and cornice, in the proportion of 4, 3, 5; the freeze ſwelling like that of the Ionic.

3. HEIGHT of the pedeſtal to be $\frac{1}{3}$ of the column.

4. HEIGHT of the capital equal to $1\frac{1}{6}$ module; of which the abaco is $\frac{1}{6}$ module, its horns projecting from the center of the column 1 module.

5. HEIGHT of the baſe $31\frac{1}{2}$ minutes, and its greateſt breadth $1\frac{1}{5}$ modules.

6. DIMINUTION of the column equal to 8 minutes.

7. IN colonades, the intercolumniation is $1\frac{1}{2}$ modules.

8. IN arches, and the columns ſet on pedeſtals,
    Diſtance of the columns from middle to middle is $7\frac{1}{4}$ modules.
    Height of the arch equal to $12\frac{1}{3}$ modules: In the clear, the height is to the ſpan as 5 to 2.
    The breadth of the pilaſters between the column and arch is $\frac{7}{10}$ modules, or 42 minutes.

IN this order the ſhaft, if fluted, is to have 24 channels and 24 fillets, one between each two flutes.

THE volutes of the capital are angular, to have the ſame appearances on every ſide, and they are drawn like thoſe in the Ionic.

The modilions in this order are worked into two faces, with an ogee between them; the breadth of the lower face $9\frac{1}{2}$ minutes, that of the upper $12\frac{1}{2}$; the diſtance of two modilions at the upper faces is 20 minutes, and at the lower faces 23 minutes; the axis of the column paſſing through the middle of a modilion.

*To draw the Mouldings in Architecture.*

The terminations or ends of flat members, are right lines.

The aſtragal, fuſarole, and torus, are terminated by a ſemicircle.

*To deſcribe the Torus.* Fig. 1. Plate I.

On AB, its breadth, deſcribe a ſemicircle.

*To make an Ovolo, whoſe breadth is* AB. Fig. 2.

Make AC $= \frac{2}{3}$ or $\frac{3}{4}$ of AB, and draw CB.

Make the angle CBD equal to the angle BCD.

Then the interſection of BD with CA will give D the center of the arc BC.

Or, Deſcribe on BC an equilateral triangle; and make the vertex the center.

The former of theſe methods is the moſt graceful.

*To make a Cavetto, whoſe breadth is* AB. Fig. 3.

Make AC $= \frac{2}{3}$ or $\frac{3}{4}$ of AB; draw BC, and produce the bottom line towards D.

Make an angle BCD equal to the angle CBD.

Then D, the interſection of CD with BD, is the center ſought.

Or, On BC deſcribe an equilateral triangle, and the vertex will be the center.

*To make a Scotia, whoſe breadth is* AB. Fig. 4.

Make AF equal to $\frac{1}{3}$ of AB.

On AF deſcribe the ſquare AC, and on BF deſcribe the ſquare BD.

Then C is the center of the arc EF, and D the center of the arc FG.

### *To make a Cima, whoſe breadth is* AB. Fig. 5.

Make AC equal to about $\frac{7}{6}$ of AB.

Draw the right line CB, which biſect in D.

On CD and DB, make iſoſceles triangles, whoſe legs DE, DF, may be each $\frac{6}{7}$ of the baſe CD, DB; and the vertexes E and F will be the centers of the arcs CD, DB.

Or, The centers of the arcs CD, DB, may be found by deſcribing equilateral triangles on the right lines CD, DB.

### *To make a Cymaiſe, or Ogee, whoſe breadth is* AB. Fig. 6.

Make AC equal to about $\frac{7}{6}$ of AB.

Draw the right line CB, which biſect in D.

Through D draw the right line EF, ſo, that the angle CDE may be equal to the angle DCE; meeting the upper and lower lines in E and F.

Then E is the center of the arc CD, and F the center of the arc DB.

### *To deſcribe the curve joining the ſhaft of a column with its upper or lower fillet, the projection of* AB *being given.* Fig. 7.

Make AC equal to twice AB.

Draw CD parallel to AB, and equal to $\frac{5}{4}$ of AC.

Then D is the center of the arc CB.

### *To draw the gradual diminution of a Column.* Fig. 8.

Draw the axis AB of the intended length of the ſhaft; and parallel thereto, at half a module diſtance, draw CD; make CE equal to half the proper diminution, and draw EF parallel to BA.

MAKE AG equal to one third of AB; and ſo high is the ſhaft to be parallel to its axis; through G draw HI at right angles to AB.

ON HI deſcribe a ſemicircumference cutting the line EF in the point 4; divide the arc H4 into equal parts at pleaſure, ſuppoſe 4; and through thoſe points draw the lines 11, 22, 33, 44.

DIVIDE the line GB into a like number of equal parts, as at the points *a*, *b*, *c*; and through theſe points draw lines parallel to IH; making *aa* = 11, *bb* = 22, *cc* = 33.

THEN a curved line drawn through the extremities H, *a*, *b*, *c*, E, will limit the gradual diminution required.

*Palladio* deſcribes another method, which is more ready in practice.

LAY a thin ruler by the points D, H, E, and the bending of the ruler will give the gradual diminution required.

*To deſcribe the Volute of the Ionic order.* Figs. 9, 10.

THE altitude AB, which is $\frac{4}{9}$ of a module, or 26$\frac{2}{3}$ minutes, is divided into 8 equal parts, *viz.* 4 from C to A, and 4 from C to B; upon CD = 3$\frac{1}{3}$, one of theſe parts, a circle is deſcribed, and called the eye of the volute, which correſponds with the aſtragal of the column.

PALLADIO gives the following manner of finding the 12 centers of the volute, which he diſcovered on an old unfiniſhed capital. Fig 9.

WITHIN the eye of the volute inſcribe a ſquare, whoſe diagonal is CD; in this ſquare draw the two diameters 13, 24, and theſe four points 1, 2, 3, 4, are the centers of the arcs A1, 1B, B3, 34, which forms the firſt revolution.

THE centers of the arcs forming the ſecond and third revolutions are thus found; ſee the eye of the volute drawn at large. Fig. 9.

Divide the radii 01, 02, 03, 04, each into 3 equal parts, as at the points 5, 6, 7, 8, 9, 10, 11, 12, and theſe will be the centers of the remaining arcs, the laſt of which is to coincide with the point c, in the eye.

Goldman obſerving that in this conſtruction the ends and beginnings of the arcs were not at right angles to the ſame radii, contrived the following conſtruction. See Fig. 10. and its eye drawn at large.

Upon one half of cd, deſcribe the ſquare 1, 2, 3, 4; and draw the lines 02, 03; divide 01, 04, each into 3 equal parts; then lines drawn through thoſe points parallel to 1, 2, their interſections with 14, 02, 03, will be centers of the volute.

So the points, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, will be the centers of the twelve arcs which together form the outward curve of the volute.

In either method, the centers of the inner curve may be thus found.

Take 0*a* equal to $\frac{7}{8}$ of 01; divide 0*a* into three equal parts, and theſe diviſions will give centers of the inner curve; the two eyes drawn at large will ſhew how the 12 inner centers are found, where they are diſtinguiſhed by large points; the 12 centers of the outward curve being marked by the figures.

In the deſcribing of theſe volutes, it will frequently happen, that the laſt quadrant will not fall on its true termination, occaſioned by the radii of the ſeveral quadrants not being exactly taken by the compaſſes: In order to avoid this inaccuracy, at leaſt in ſome degree, here is ſubjoined a table ſhewing the length of each radius, computed from *Goldman*'s method: But it may alſo be applied to *Palladio*'s, the radius of the largeſt quadrant not differing $\frac{3}{100}$ of a minute, or $\frac{1}{2000}$ of a module from the truth; and excepting the arc deſcribed from the firſt center, the reſt may be made quadrants in the ſame manner as ſhewn in *Goldman*'s method.

## A TABLE *of the lengths, in minutes, of the ſeveral radii of the outward and inner volutes.*

| N° Rad. | Outward Curve. | Inward Curve. | In parts of 1ſt rad. | |
|---|---|---|---|---|
| | | | Outward. | Inward. |
| 1 | $\frac{85}{6}$ = 14,166 | $\frac{605}{48}$ = 12,604 | 100,000 | 88,969 |
| 2 | $\frac{75}{6}$ = 12,500 | $\frac{535}{48}$ = 11,146 | 88,235 | 78,677 |
| 3 | $\frac{65}{6}$ = 10,833 | $\frac{465}{48}$ = 9,687 | 76,468 | 68,379 |
| 4 | $\frac{55}{6}$ = 9,166 | $\frac{395}{48}$ = 8,229 | 64,705 | 58,087 |
| 5 | $\frac{70}{9}$ = 7,777 | $\frac{1010}{144}$ = 7,014 | 54,901 | 49,510 |
| 6 | $\frac{60}{9}$ = 6,666 | $\frac{870}{144}$ = 6,041 | 47,058 | 42,642 |
| 7 | $\frac{50}{9}$ = 5,555 | $\frac{730}{144}$ = 5,069 | 39,215 | 35,781 |
| 8 | $\frac{40}{9}$ = 4,444 | $\frac{590}{144}$ = 4,097 | 31,372 | 28,920 |
| 9 | $\frac{65}{18}$ = 3,611 | $\frac{485}{144}$ = 3,368 | 25,490 | 23,774 |
| 10 | $\frac{55}{18}$ = 3,055 | $\frac{415}{144}$ = 2,882 | 21,568 | 20,343 |
| 11 | $\frac{45}{18}$ = 2,500 | $\frac{345}{144}$ = 2,395 | 17,647 | 16,906 |
| 12 | $\frac{35}{18}$ = 1,944 | $\frac{275}{144}$ = 1,909 | 13,725 | 13,475 |

To uſe this table, a ſcale of $\frac{1}{4}$ of a module ſhould be made, and divided into *15* minutes, and the extream diviſion decimally divided, whereby the lengths of the ſeveral radii may be taken: But as the ſector is an univerſal ſcale, there are two other columns added, applicable to the ſector; where the longer radius 14,166 is made a tranſverſe diſtance to 10 and 10, or 100 and 100, on the line of lines, and all the other radii of both curves are proportioned thereto: Now the centers of the curves being found as ſhewn in the eyes of the volute, the ſeveral radii may be taken from the ſector, and the curves more accurately deſcribed than by any other method.

### *To deſcribe the Flutings and Fillets in channelled columns.* Fig. 11.

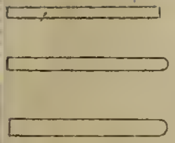
In the Doric, the circumference of the column being divided into 20 equal parts (here the $\frac{1}{4}$ circumference is divided into 5), of which *ab* is one; on *ab* deſcribe a ſquare, and the center *c* of that ſquare is the center of the channel or flute required.

In the Ionic, and Corinthian, divide the circumference of the column into 24 equal parts (here the $\frac{1}{4}$ circumference is divided into 6), of which *ad* is one; divide *ad* into 4 equal parts; then $ae = \frac{3}{4}ad$ is the breadth of the flute, and $ed = \frac{1}{4}ad$ is the breadth of the fillet.

The flutes are ſemicircles deſcribed on the chords of their arcs in the column.

In the three following tables are contained the heights and projections of the parts of each order, according to the proportions given by *Palladio*; the orders of this architect were choſen, becauſe the *Engliſh*, at preſent, are more fond of copying his productions, than thoſe of any other architect.

The firſt table ſerves for the pedeſtal, the ſecond for the column, and the third for the entablature, of each order. Each table is divided into ſeven principal columns: In the firſt, beginning at the left hand, is contained the names of the primary diviſions; in the ſecond thoſe of the ſeveral diviſions and members in the orders; and the other five, titled with *Tuſcan*, *Doric*



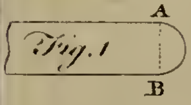
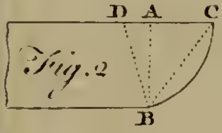


Fig. 1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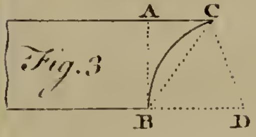


Fig. 3

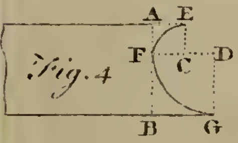


Fig. 4

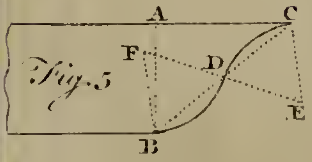
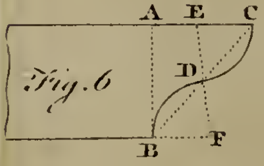


Fig. 5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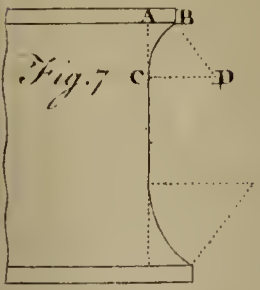


Fig. 7

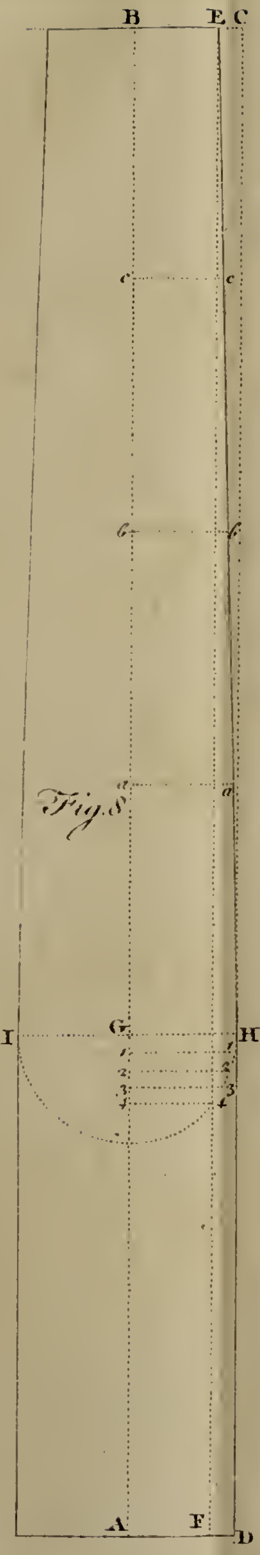


Fig. 8

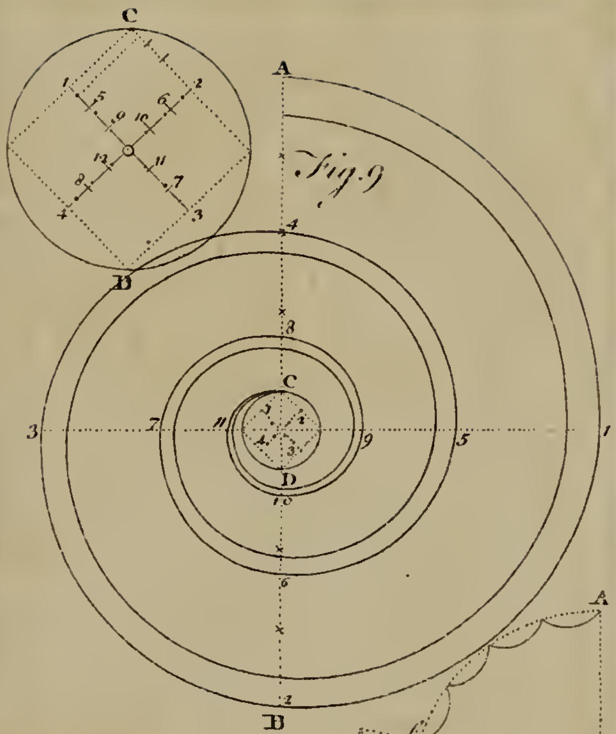


Fig. 9

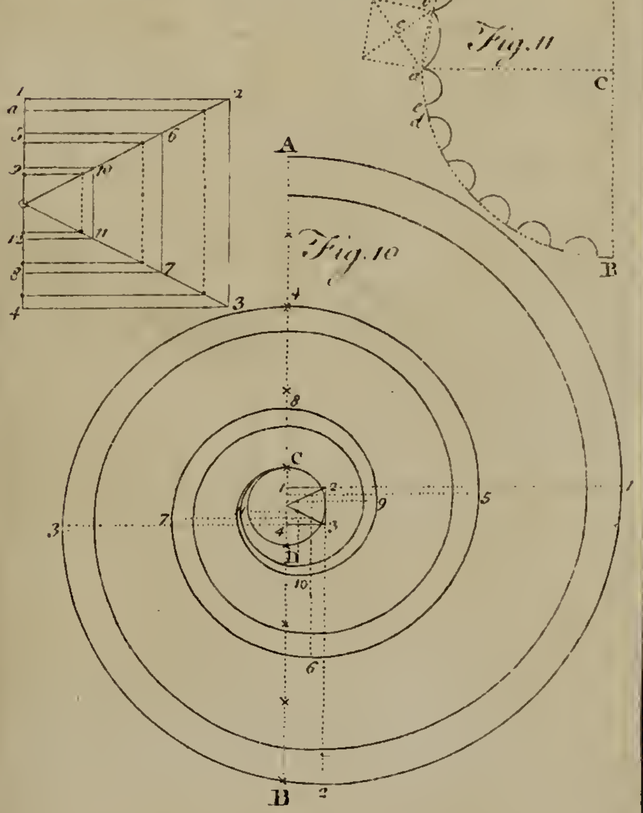


Fig. 10
Fig. 11

B. Cole Sculp.

*Doric*, *Ionic*, *Corinthian*, *Roman*, contain the numbers expreſſing the altitudes, and projections taken from the axis, or middle of the column, of the ſeveral members belonging to their correſponding orders.

THE column containing each order, is divided, firſt into two other columns, one ſhewing the altitudes, and ſigned Alt. and the other, the projections, and ſigned Proj. Each of theſe is alſo divided into two other columns, one containing modules, and marked Mo. and the other, the minutes and parts, and marked Mi.

UNDER the table of the pedeſtal there is another table, ſhewing the general proportions for the heights of the orders.

IN each of the orders of architecture, the height of the order, and the diameter of the column, have a conſtant relation to one another.

THEREFORE, if the diameter of the column be given, the height of the order is given alſo: And having determined by what ſcale the order is to be drawn, ſuch as $\frac{1}{2}$ inch, 1 inch, 2 inches, &c. to a foot or yard, &c. Take from ſuch ſcale, the part or parts expreſſing the diameter of the column, and make this extent a tranſverſe diſtance to 6 and 6 (*i. e.* 60 and 60) on the ſcales of lines, and the ſector will be opened ſo, that the ſeveral proportions of the order may be taken from it.

EXAM. *Suppoſe the diameter of a column is to be* 18 *inches*; *and the drawing of the order is to be delineated from a ſcale of an inch to a foot: that is, the diameter of the column in the drawing is to be an inch and half.*

MAKE the tranſverſe diſtance of 6 and 6, on the ſcales of lines, equal to $1\frac{1}{2}$ inch, and the ſector is fitted for the ſcale.

IF the height of the order is given, divide this height, by the height of the order in the table; and the quotient will be the diameter of the column.

Exam. *What muſt be the diameter of the column in the* Ionic *order, when the whole height of the order is fixed at* 18 *feet* 6 *inches.*

The height of the order in the table is 13 mo. $29\frac{1}{4}$ mi. $= 13\frac{29,25}{60} = 13,4875$ modules: And 18 f. 6 in. $= 18,5$ feet. Therefore $\frac{18,5}{13,4875} = 1,3709$ feet $= 1$ f. $4\frac{1}{2}$ inches nearly: And the ſector may be fitted to this, as before directed, according to the intended ſize of the draught.

## *To delineate an Order by theſe Tables.*

Having determined the diameter of the column at bottom, and ſet the ſector to the intended ſcale, draw a line to repreſent the axis or middle of the order.

On this line, lay the parts for the heights of the pedeſtal, column, and entablature, taken from the table of general proportions.

Within each of theſe parts reſpectively, lay the ſeveral altitudes taken from the tables of particulars, under the word Alt. Through each of the points marked on the axis, draw lines perpendicular to the axis, or draw one line perpendicular, and the others parallel thereto.

On the lines drawn perpendicular to the axis, lay the projections correſponding to the reſpective altitudes; theſe projections are to be laid on both ſides of the axis, for the pedeſtal and column; and only on one ſide, for the entablature, join the extremities of the projections with ſuch lines as are proper to expreſs the reſpective mouldings and parts: And the order, excluſive of its ornaments, will be delineated.

As the altitudes of many of the parts are very ſmall, it will not be convenient, if poſſible, to take from the ſcale of lines, ſuch ſmall parts alone; therefore it may be beſt to proceed as in the following example of the *Ionic* order.

*To conſtruct the Pedeſtal.* Plate II.

In the line AD, which repreſents the axis of the order, take the baſe $Aa = 42\frac{1}{2}$ min., the die $ad = 1$ mod. K 35 min.; and the capital $dD = 22\frac{3}{4}$ min. Then to draw the ſmall members in the baſe and cornice, proceed thus.

To the minutes in the baſe, $42\frac{1}{2}$, add ſome even number of minutes, ſuppoſe $30 = aB$, and the ſum $72\frac{1}{2}$ is equal to AB; then compoſe a table, ſuch as the following one, wherein the alt. of the plinth is ſubtracted out of the No. $72\frac{1}{2}$; then the torus out of this remainder; then the cymа out of this remainder; then the fillet out of this; and laſtly, the cavetto out of this remainder. Thus,

| | | | Min. |
|---|---|---|---|
| Baſe with 30 minutes | | . . . . . . . . . . . . | $72\frac{1}{2}$ |
| This leſs by the plinth, | $28\frac{1}{2}$, | remains . . . | 44 |
| This leſs by the torus, | 4, | remains . . . | 40 |
| This leſs by the fillet, | $0\frac{3}{4}$, | remains . . | $39\frac{1}{4}$ |
| This leſs by the cyma, | 5, | remains . . . | $34\frac{1}{4}$ |
| This leſs by the fillet, | $0\frac{3}{4}$, | remains . . | $33\frac{1}{2}$ |
| This leſs by the cavetto, | $3\frac{1}{2}$, | remains . . . | 30, |

the minutes firſt added.

Then the ſeveral numbers in the table may be taken from the line of lines on the ſector, and applied from B towards A. Thus,

Make $B1 = 44$, $B2 = 40$, $B3 = 39\frac{1}{4}$, $B4 = 34\frac{1}{4}$, $B3 = 33\frac{1}{4}$; draw lines through theſe points at right angles to AD, and on theſe lines lay the reſpective projections, as ſhewn in the general table; then the proper curvature or figure being drawn at the extremities of the numbers, the baſe of the pedeſtal will be made.

It will be found moſt convenient to lay off the numbers from the greater to the leſſer ones; for then there is only one motion required in the joints of the compaſſes, which is, to bring them cloſer and cloſer every diſtance laid down.

And in the ſame manner, for the cornice of the pedeſtal, take a point C, 30 minutes below the cornice; and tabulate as before.

Cornice with 30 min. . . . . . . . . . . $52\frac{3}{4}$ = CD
This leſs by the fillet or cap, $2\frac{1}{2}$, leaves $50\frac{1}{4}$ = c1
Ditto . . . . . ogee . . . $3\frac{1}{2}$, ditto $46\frac{3}{4}$ = c2
Ditto . . . . . corona . . $4\frac{1}{2}$ . . . $42\frac{1}{4}$ = c3
Ditto . . . . . fillet . . . $1\frac{3}{4}$ . . . $40\frac{1}{2}$ = c4
Ditto . . . . . cyma . . . $5\frac{1}{4}$ . . . $35\frac{1}{4}$ = c5
Ditto . . . . . fillet . . . $1\frac{3}{4}$ . . . $33\frac{1}{2}$ = c6
Ditto . . . . . cavetto . . $3\frac{1}{2}$ . . . 30 = c*d*.

These numbers laid from c towards d, gives the altitudes of the members of the cornice.

In like manner the mouldings about the baſe and capital are laid down, by taking 30 minutes in the ſhaft both above the baſe and below the capital; having firſt ſet on the axis, the reſpective heights of the baſe, ſhaft, and capital,

*Thus for the Baſe.*

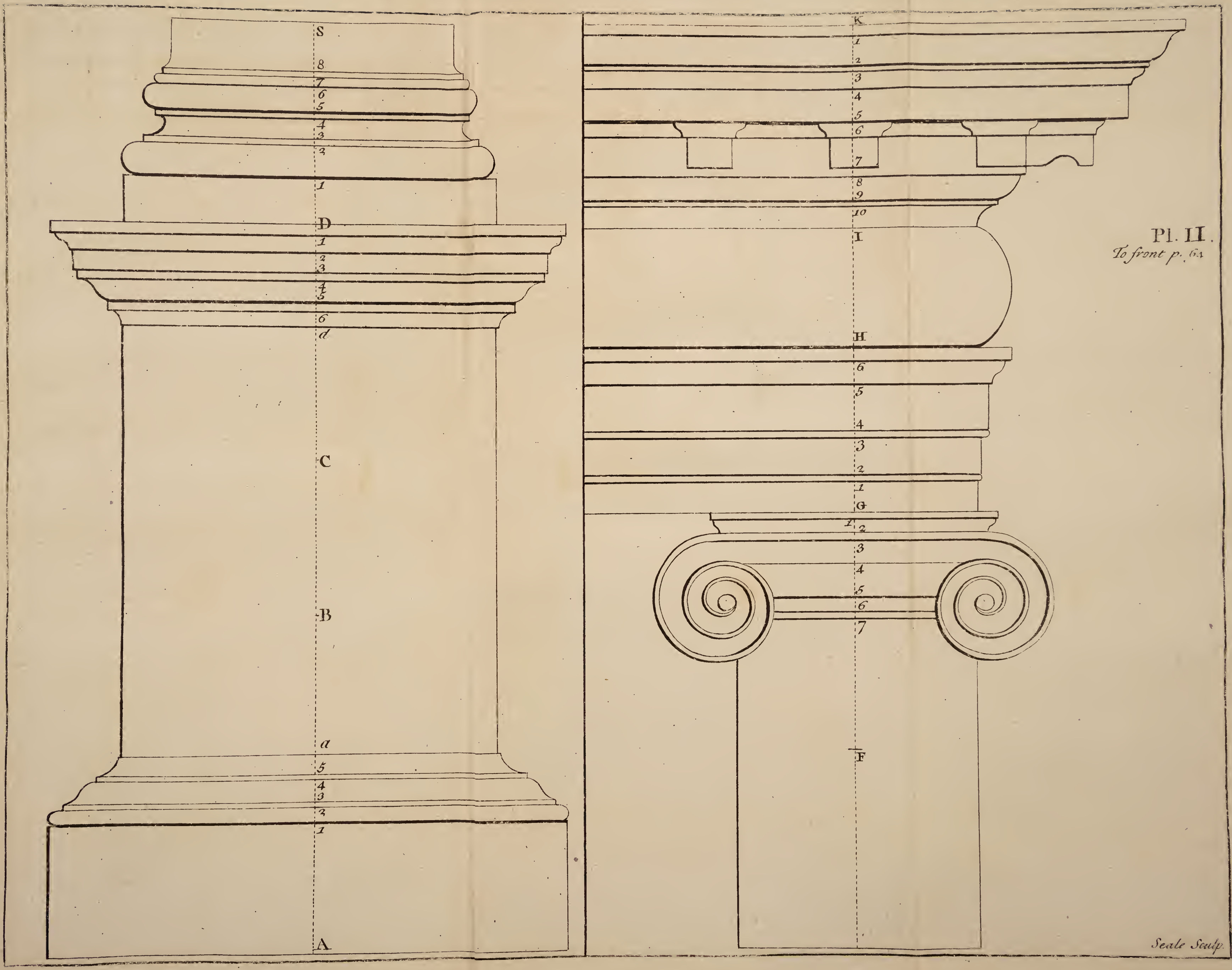
The baſe $33\frac{1}{2}$ min. with 30 added = $63\frac{1}{2}$ = sd
This leſs by the plinth 10 min. leaves $53\frac{1}{2}$ = s1
Ditto . . . . . torus $7\frac{1}{2}$ . . . . . 46 = s2
Ditto . . . . . fillet $1\frac{1}{4}$ . . . . . $44\frac{3}{4}$ = s3
Ditto . . . . . ſcotia $4\frac{2}{3}$ . . . . . $40\frac{1}{12}$ = s4
Ditto . . . . . fillet $1\frac{1}{4}$ . . . . . $38\frac{5}{6}$ = s5
Ditto . . . . . torus $5\frac{1}{3}$ . . . . . $33\frac{1}{2}$ = s6
Ditto . . . . . aſtragal $2\frac{1}{4}$ . . . . . $31\frac{1}{4}$ = s7
Ditto . . . . . fillet $1\frac{1}{4}$ . . . . . 30 = s8.

s8 is here ſuppoſed to be 30, though the plate is not high enough to admit 30 minutes to be laid in the ſhaft of the column.

*For the Capital.*

The capital $24\frac{1}{4}$ with 30 added, gives $54\frac{1}{4}$ = fg
This leſs by the plateband $1\frac{3}{4}$, leaves $52\frac{1}{2}$ = f1
Ditto . . . . . . . ogee . . $3\frac{1}{3}$ . . . $49\frac{1}{6}$ = f2

This



Pl. II.

*To front p. 61*

*Seale Sculp.*

This leſs by the rim of volute $1\frac{1}{3}$, leaves $47\frac{5}{6}$ = F3
Ditto . . . . . . . hollow $5\frac{1}{3}$ . . . $42\frac{1}{2}$ = F4
Ditto . . . . . . . ovolo . . $7\frac{1}{2}$ . . . 35 = F5
Ditto . . . . . . . aſtragal $3\frac{1}{3}$ . . . $31\frac{2}{3}$ = F6
Ditto . . . . . . . fillet . . $1\frac{2}{3}$ . . . 30 = F7.

### *To conſtruct the Cornice.*

IN the axis take GH = 36 for the architrave, HI = 27 for the freeze, and IK = 46 for the cornice. Then,

### *For the parts of the Architrave.*

To the freeze 27 add HG 36, gives 63 = IG
This leſs by the firſt face $6\frac{1}{2}$, leaves $56\frac{1}{2}$ = I1
Ditto . . . . . . fuſarole $1\frac{1}{4}$ . . . $55\frac{1}{4}$ = I2
Ditto . . . . . . 2d face . $8\frac{1}{3}$ . . . $46\frac{11}{12}$ = I3
Ditto . . . . . . fuſarole 2 . . . $44\frac{11}{12}$ = I4
Ditto . . . . . . 3d face $10\frac{1}{2}$ . . . $34\frac{5}{12}$ = I5
Ditto . . . . . . ogee . . $4\frac{3}{4}$ . . . $29\frac{2}{3}$ = I6
Ditto . . . . . . fillet . . $2\frac{2}{3}$ . . . 27 = IH.

### *For the Cornice.*

To the freeze 27 add the cornice 46, gives 73 = HK
This leſs by the fillet . . . $2\frac{1}{2}$, leaves $70\frac{1}{2}$ = H1
Ditto . . . . . cima . 7 . . . . $63\frac{1}{2}$ = H2
Ditto . . . . . fillet . . 1 . . . . $62\frac{1}{2}$ = H3
Ditto . . . . . ogee . . $3\frac{1}{2}$ . . . . 59 = H4
Ditto . . . . . corona 8 . . . . 51 = H5
Ditto . . . . . ogee . . 3 . . . . 48 = H6
Ditto . . . . . modilion $7\frac{1}{2}$ . . . . $40\frac{1}{2}$ = H7
Ditto . . . . . fillet . . $1\frac{1}{2}$ . . . . 39 = H8
Ditto . . . . . ovolo . . 6 . . . . 33 = H9
Ditto . . . . . fillet . . 1 . . . . . . 32 = H10
Ditto . . . . . cavetto . 5 . . . . . 27 = HI.

TABLES may be made in like manner for either of the orders, to be taken from the ſector: The projections from the axis being all of them large numbers, they may be taken from the ſector eaſily enough after it is ſet to the diameter of the column, as before ſhewn.

A LITTLE reflection will make this very clear, and perhaps more ſo, than by beſtowing more words thereon.

A TABLE *ſhewing the Altitudes and Projections of*
*Order; according to the*

| | Names of the Members. | Tuſcan. | | | | Doric. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Alt. | | Proj. | | Alt. | | Proj. | |
| | | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. |
| CORNICE. | Fillet | — | — | — | — | 0 | $3\frac{1}{3}$ | 0 | 56 |
| | Ogee | — | — | — | — | — | — | — | — |
| | Corona | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | — | — | — | — |
| | Cima | — | — | — | — | 0 | 9 | — | — |
| | Fillet | — | — | — | — | 0 | $1\frac{1}{4}$<br>$1\frac{1}{4}$ | 0 | 47<br>$45\frac{3}{4}$ |
| | Aſtragal | — | — | — | — | — | — | — | — |
| | Ogee | — | — | — | — | — | — | — | — |
| | Cavetto | — | — | — | — | 0 | 5 | 0 | $41\frac{1}{4}$ |
| | Fillet | — | — | — | — | — | — | — | — |
| | The Cornice | — | — | — | — | 0 | $26\frac{1}{6}$ | — | — |
| | THE DIE | 1 | 0 | 0 | 42 | 1 | 20 | 0 | 40 |
| | The Baſe | — | — | — | — | 0 | 40 | — | — |
| BASE. | Fillet | — | — | — | — | — | — | — | — |
| | Cavetto | — | — | — | — | 0 | 5 | 0 | $41\frac{1}{4}$ |
| | Ogee | — | — | — | — | — | — | — | — |
| | Aſtragal | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | 0 | $1\frac{1}{4}$<br>$1\frac{1}{4}$ | 0 | 46<br>$47\frac{1}{4}$ |
| | Cima | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | — | — | — | — |
| | Torus | — | — | — | — | — | 5 | 0 | 50 |
| | Plinth | — | — | — | — | — | $27\frac{1}{2}$ | 0 | 50 |

A TABLE *of general*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| The Order | 9 | $44\frac{1}{2}$ | — | — | 12 | $15\frac{1}{6}$ | — | — |
| The Entablature | 1 | $44\frac{1}{2}$ | — | — | 1 | 55 | — | — |
| The Column | 7 | 0 | — | — | 8 | 0 | — | — |
| The Pedeſtal | 1 | 0 | — | — | 2 | $20\frac{5}{6}$ | — | — |

# FIRST.

*every Moulding and Part in the Pedeſtals of each Proportions given by* Palladio.

| Ionic. | | | | Corinthian. | | | | Roman. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Alt. | | Proj. | | Alt. | | Proj. | | Alt. | | Proj. | |
| Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. |
| 0 | 2½ | 0 | 56¼ | 0 | 2½ | 0 | 57 | 0 | 2½ | 0 | 57 |
| 0 | 3½ | 0 | 55¼<br>53¼ | 0 | 3½ | 0 | 56<br>54¼ | 0 | 3½ | 0 | 56<br>54¼ |
| 0 | 4½ | 0 | 52¼ | 0 | 4¼ | 0 | 53¼ | 0 | 5½ | 0 | 53½ |
| 0 | 1¾ | 0 | 51¾ | — | — | — | — | 0 | 1 | 0 | 52¾ |
| 0 | 5¼ | — | — | 0 | 4¼ | 0 | 49¼<br>46 | 0 | 8½ | — | — |
| 0 | 1¾ | 0 | 44¾ | 0 | 0¾ | 0 | 46 | — | — | — | — |
| — | — | — | — | — | — | — | — | 0 | 3 | 0 | 46¼ |
| — | — | — | — | 0 | 3¾ | 0 | 45<br>43 | — | — | — | — |
| 0 | 3½ | 0 | 41¾ | — | — | — | — | — | — | — | — |
| — | — | — | — | — | — | — | — | 0 | 1¾ | 0 | 44¾ |
| 0 | 22¾ | — | — | 0 | 19 | — | — | 0 | 25¼ | — | — |
| 1 | 35 | 0 | 41¼ | 1 | 36 | 0 | 42 | 2 | 6½ | 0 | 42 |
| 0 | 42½ | — | — | 0 | 38 | — | — | 0 | 50 | — | — |
| — | — | — | — | — | — | — | — | 0 | 1 | 0 | 45½ |
| 0 | 3½ | 0 | 41¾ | — | — | — | — | — | — | — | — |
| — | — | — | — | 0 | 4 | 0 | 43<br>46 | — | — | | — |
| — | — | — | — | — | — | — | — | 0 | 3 | 0 | 47 |
| 0 | 0¾ | 0 | 47¼ | 0 | 0¾ | 0 | 47 | — | — | — | — |
| 0 | 5 | — | — | 0 | 5 | — | — | 0 | 7½ | 0 | 45¼<br>54¾ |
| 0 | 0¾ | 0 | 53¾ | 0 | 0¾ | 0 | 55 | 0 | 1 | 0 | 54¾ |
| 0 | 4 | 0 | 56¼ | 0 | 4 | 0 | 57 | 0 | 4½ | 0 | 57 |
| 0 | 28½ | 0 | 56¼ | 0 | 23½ | 0 | 57 | 0 | 33 | 0 | 57 |

*Proportions for the Orders.*

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | 29¼ | — | — | 13 | 27 | — | — | 15 | 22¼ | — | — |
| 1 | 49 | — | — | 1 | 54 | — | — | 2 | 0 | — | — |
| 9 | 0 | — | — | 9 | 30 | — | — | 10 | 0 | — | — |
| 2 | 40¼ | — | — | 2 | 33 | — | — | 3 | 22½ | — | — |

## *A* TABLE, *ſhewing the Altitudes and Projections of according to the Propor-*

| | Names of the Members. | Tuſcan. | | | | Doric. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Alt. | | Proj. | | Alt. | | Proj. | |
| | | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. |
| CAPITAL. | Angular Volutes | — | — | — | — | — | — | — | — |
| | Abacus Ovolo | — | — | — | — | — | — | — | — |
| | Abacus Fillet | — | — | — | — | 0 | 1¼ | 0 | 38¾ |
| | Abacus Cavetto | — | — | — | — | — | — | — | — |
| | Baſket Rim | — | — | — | — | — | — | — | — |
| | Ogee | — | — | — | — | 0 | 2½ | 0 | 37¾<br>36¼ |
| | Abacus | 0 | 10 | 0 | 30 | 0 | 6¼ | 0 | 35¾ |
| | Volute fillet or rim | — | — | — | — | — | — | — | — |
| | Volute chan. or hollow | — | — | — | — | — | — | — | — |
| | Ovolo | 0 | 10 | 0 | 29 | 0 | 6½ | 0 | 34⅓ |
| | Aſtragal | — | — | — | — | — | — | — | — |
| | Fillet | 0 | 1½ | 0 | 24½ | 0<br>0<br>0 | 1 1/9<br>1 1/9<br>1 1/9 | 0<br>0<br>0 | 29¾<br>28½<br>27¼ |
| | Collarino | 0 | 8½ | 0 | 22½ | 0 | 10 | 0 | 26 |
| | Middle Volute | — | — | — | — | — | — | — | — |
| | Courſ. of leaves, folding half their height 3d | — | — | — | — | — | — | — | — |
| | 2d | — | — | — | — | — | — | — | — |
| | 1ſt | — | — | — | — | — | — | — | — |
| SHAFT. | Aſtragal | 0 | 4 | 0 | 27 | 0 | 3½ | 0 | 30 |
| | Fillet | 0 | 1½ | 0 | 24½ | 0 | 1½ | 0 | 28¼ |
| | Body of the Column | 5 | 54½ | 0 | 22½<br>30 | 6 | 53¾ | 0 | 26<br>30 |
| | Fillet | 0 | 2½ | 0 | 33¾ | 0 | 1¼ | 0 | 33½ |
| | Aſtragal | — | — | — | — | — | — | — | — |
| BASE. | Torus | — | — | — | — | 0 | 5½ | 0 | 36⅔ |
| | Aſtragal | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | 0 | 1¼ | 0 | 35 |
| | Scotia | — | — | — | — | 0 | 4½ | 0 | 33⅓ |
| | Fillet | — | — | — | — | 0 | 1¼ | 0 | 36⅔ |
| | Aſtragal | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | — | — | — | — |
| | Scotia | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | — | — | — | — |
| | Torus | 0 | 12½ | 0 | 40 | 0 | 7½ | 0 | 40 |
| | Plinth | 0 | 15 | 0 | 40 | 0 | 10 | 0 | 40 |
| | Baſe | 0 | 27½ | — | — | 0 | 30 | — | — |
| | Shaft | 6 | 2½ | — | — | 7 | 0 | — | — |
| | Capital | 0 | 30 | — | — | 0 | 30 | — | — |

*every Moulding and Part in the Columns of each Order;*
*tions given by* Palladio.

| Ionic. | | | | Corinthian. | | | | Roman. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Alt. | | Proj. | | Alt. | | Proj. | | Alt. | | Proj. | |
| Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. |
| 0 | 26⅔ | 0 | 41⅔ | 0 | 12 | 0 | 41 | 0 | 25⅔ | 0 | 35 |
| — | — | — | — | 0 | 3 | 0 | 45 | 0 | 3 | 0 | 44 |
| 0 | 1¾ | 0 | 31½ | 0 | 1⅓ | 0 | 42 | 0 | 1⅓ | 0 | 42½ |
| — | — | — | — | 0 | 5⅔ | 0 | 39 | 0 | 5⅔ | 0 | 41 |
| — | — | — | — | 0 | 2½ | — | — | — | — | — | — |
| 0 | 3⅓ | 0 | 30¾<br>29 | — | — | — | — | — | — | — | — |
| — | — | — | — | — | — | — | — | — | — | — | — |
| 0 | 1⅓ | — | — | — | — | — | — | — | — | — | — |
| 0 | 5⅓ | — | — | — | — | — | — | — | — | — | — |
| 0 | 7½ | 0 | 35 | — | — | — | — | 0 | 5½ | 0 | 32 |
| — | — | — | — | — | — | — | — | 0 | 3 | 0 | 26 |
| — | — | — | — | — | — | — | — | — | — | — | — |
| — | — | — | — | — | — | — | — | 0 | 1½ | 0 | 24 |
| — | — | — | — | — | — | — | — | — | — | — | — |
| — | — | — | — | — | — | — | — | — | — | — | — |
| — | — | — | — | 0 | 9½ | — | — | — | — | — | — |
| — | — | — | — | 0 | 8 | — | — | 0 | 10 | — | — |
| — | — | — | — | 0 | 20 | 0 | 41 | 0 | 20 | 0 | 39 |
| — | — | — | — | 0 | 20 | 0 | 35 | 0 | 20 | 0 | 35 |
| 0 | 3⅓ | 0 | 30 | 0 | 3⅔ | 0 | 30 | 0 | 4 | 0 | 30 |
| 0 | 1⅔ | 0 | 28⅓ | 0 | 1⅓ | 0 | 28 | 0 | 1½ | 0 | 28 |
| 8 | 2¼ | 0 | 26<br>30 | 7 | 40¾ | 0 | 26<br>30 | 8 | 9 | 0 | 26<br>30 |
| 0 | 1¼ | 0 | 33 | 0 | 1¾ | 0 | 33½ | 0 | 1 | 0 | 34 |
| 0 | 2¼ | 0 | 34½ | 0 | 2½ | 0 | 35½ | 0 | 3 | 0 | 35½ |
| 0 | 5⅓ | 0 | 37 | 0 | 5 | 0 | 37½ | 0 | 4½ | 0 | 37 |
| — | — | — | — | 0 | 1½ | 0 | 35½ | — | — | — | — |
| 0 | 1¼ | 0 | 34½ | 0 | 0⅔ | 0 | 34 | 0 | 0⅔ | 0 | 35½ |
| 0 | 4⅔ | — | — | 0 | 3¾ | — | — | 0 | 3 | — | — |
| 0 | 1¼ | 0 | 37 | 0 | 0¾ | 0 | 37 | 0 | 0½ | 0 | 36½ |
| — | — | — | — | 0 | 1¾ | 0 | 38½ | 0 | 1 | 0 | 37 |
| | | | | | | | | 0 | 1 | 0 | 37 |
| — | — | — | — | — | — | — | — | 0 | 0½ | 0 | 36½ |
| — | — | — | — | — | — | — | — | 0 | 3 | — | — |
| — | — | — | — | — | — | — | — | 0 | 0⅔ | 0 | 38½ |
| 0 | 7½ | 0 | 41¼ | 0 | 7 | 0 | 42 | 0 | 7 | 0 | 42 |
| 0 | 10 | 0 | 41¼ | 0 | 9⅔ | 0 | 42 | 0 | 9⅔ | 0 | 42 |
| 0 | 30 | — | — | 0 | 30 | — | — | 0 | 1½ | — | — |
| 8 | 10[illegible] | — | — | 7 | | — | — | 8 | 18½ | — | — |
| 0 | 19¼ | — | — | 1 | 10 | — | — | 1 | 10 | — | — |

A TABLE, *ſhewing the Altitudes and Projections of*
*Order; according to the Pro-*

| | Names of the Members. | Tuſcan. | | | | Doric. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Alt. | | Proj. | | Alt. | | Proj. | |
| | | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo. | Mi. |
| CORNICE. | Fillet | 0 | 3½ | 1 | 6 | 0 | 2¼ | 1 | 16 |
| | Cima | 0 | 10 | — | — | 0 | 6 | — | — |
| | Fillet | 0 | 2 | 0 | 54¼ | 0 | 0¾ | 1 | 8 |
| | Ogee | — | — | — | — | 0 | 3¼ | 1<br>1 | 7<br>5½ |
| | Corona | 0 | 10 | 0 | 52¼ | 0 | 8 | 1 | 4½ |
| | Ovolo | 0 | 9 | 0 | 42 | 0 | 6 | 0 | 39½ |
| | Fillet or Aſtragal | 0 | 1½ | 0 | 32 | 0 | 1 | 0 | 35½ |
| | Ogee | — | — | — | — | — | — | — | — |
| | Modilion 2d Face | — | — | — | — | — | — | — | — |
| | Modilion Ogeee | — | — | — | — | — | — | — | — |
| | Modilion 1ſt Face | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | — | — | — | — |
| | Ovolo | — | — | — | — | — | — | — | — |
| | Ogee | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | — | — | — | — |
| | Dentel | — | — | — | — | — | — | — | — |
| | Aſtragal | — | — | — | — | — | — | — | — |
| | Fillet | — | — | — | — | — | — | — | — |
| | Ogee | — | — | — | — | — | — | — | — |
| | Cavetto | 0 | 7½ | 0 | 23½ | 0 | 5 | 0 | 31 |
| | Trigliphs Capital | — | — | — | — | 0 | 5 | 0 | 30½ |
| | The Cornice | 0 | 43½ | — | — | 0 | 38 | — | — |
| | THE FREEZE | 0 | 26 | 0 | 22½ | 0 | 45 | 0 | 26 |
| | The Architrave | 0 | 35 | — | — | 0 | 30 | — | — |
| ARCHITRAVE. | Fillet | 0 | 5 | 0 | 27½ | 0 | 4½ | 0 | 28 |
| | Cavetto | — | — | — | — | — | — | — | — |
| | Ogee | — | — | — | — | — | — | — | — |
| | Aſtragal or Fuſarole | — | — | — | — | — | — | — | — |
| | Third Face | — | — | — | — | — | — | — | — |
| | Aſtragal or Fuſarole | — | — | — | — | — | — | — | — |
| | Second Face | 0 | 17½ | 0 | 24 | 0 | 14½ | 0 | 27 |
| | Ogee | — | — | — | — | — | — | — | — |
| | Aſtragal or Fuſarole | — | — | — | — | — | — | — | — |
| | Firſt Face | 0 | 12½ | 0 | 22½ | 0 | 11 | 26 | 26 |

*every Moulding and Part in the Entablature of each*
*portions given by* Palladio.

| Ionic. | | | | Corinthian. | | | | Roman. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Alt. | | Proj. | | Alt. | | Proj. | | Alt. | | Proj. | |
| Mo. | Mi. | Mo. | Mi. | Mo. | Mi. | Mo | Mi. | Mo. | Mi. | Mo. | Mi. |
| 0 | 2½ | 1 | 12 | 0 | 2¼ | 1 | 14 | 0 | 2½ | 1 | 18½ |
| 0 | 7 | — | — | 0 | 6¼ | — | — | 0 | 8 | — | — |
| 0 | 1 | 1 | 4 | 0 | 0⅔ | 1 | 6½ | 0 | 1 | 1 | 10 |
| 0 | 3½ | 1<br>0 | 3<br>0½ | 0 | 3 | 1<br>1 | 5½<br>4 | 0 | 3¾ | 1<br>1 | 9<br>6 |
| 0 | 8 | 0 | 59½ | 0 | 7⅓ | 1 | 3 | 0 | 9½ | 1 | 5 |
| — | — | — | — | — | — | — | — | 0 | 2½ | 0 | 55 |
| — | — | — | — | 0 | 0⅔ | 1 | 2 | 0 | 1¼ | 0 | 54 |
| 0 | 3 | 0 | 55<br>53 | 0 | 2⅓ | 1<br>0 | 1<br>59 | — | — | — | — |
| | — | — | — | — | — | — | — | 0 | 6½ | 0 | 53 |
| 0 | 7½ | 0 | 52 | 0 | 7¼ | 0 | 40¼ | 0 | 1¾ | 0 | 52½ |
| | — | — | — | — | — | — | — | 0 | 3¼ | 0 | 51 |
| 0 | 1½ | 0 | 37 | 0 | 1 | 0 | 40 | 0 | 1 | 0 | 51 |
| 0 | 6 | 0 | 36 | 0 | 4½ | 0 | 39 | — | — | — | — |
| — | — | — | — | — | — | — | — | 0 | 5 | 0 | 35½<br>29 |
| — | — | — | — | 0 | 1 | 0 | 36 | — | — | — | — |
| — | — | — | — | 0 | 5½ | 0 | 35 | — | — | — | — |
| — | — | — | — | — | — | — | — | 0 | 2 | 0 | 30 |
| 0 | 1 | 0 | 31½ | 0 | 1 | 0 | 32 | 0 | 2 | 0 | 28½ |
| — | — | — | — | 0 | 4½ | 0 | 31<br>27 | — | — | — | — |
| 0 | 5 | 0 | 27 | — | — | — | — | — | — | — | — |
| — | — | — | — | — | — | — | — | — | — | — | — |
| 0 | 46 | — | — | 0 | 47½ | — | — | 0 | 50 | — | — |
| 0 | 27 | 0 | 34 | 0 | 28½ | 0 | 26 | 0 | 30 | 0 | 35 |
| 0 | 30 | — | — | 0 | 38 | — | — | 0 | 40 | — | — |
| 0 | 2⅔ | 0 | 34 | 0 | 2½ | 0 | 34½ | 0 | 2⅙ | 0 | 35 |
| — | — | — | — | — | — | — | — | 0 | 4⅛ | 0 | 32 |
| 0 | 4¾ | 0 | 33<br>30 | 0 | 5 | 0 | 33½<br>30 | 0 | 3⅔ | 0 | 31<br>29 |
| — | — | — | — | 0 | 2 | 0 | 29½ | — | — | — | — |
| 0 | 10½ | 0 | 29 | 0 | 10½ | 0 | 28 | — | — | — | — |
| 0 | 2 | 0 | 29 | 0 | 1¾ | 0 | 28 | 0 | 1 5/12 | — | 29 |
| 0 | 8½ | 0 | 27½ | 0 | 8¼ | 0 | 27 | 0 | 1 | 0 | 28 |
| — | — | — | — | — | — | — | — | 0 | 2⅔ | 0 | 27½<br>26½ |
| 0 | 1¼ | 0 | 27½ | 0 | 1¾ | 0 | 27 | — | — | — | — |
| 0 | 6½ | 0 | 26½ | 0 | 6½ | 0 | 26 | 0 | 11 | 0 | 26 |

## SECT. XIII.

### *Some Uſes of the Scales of Polygons.* Pl. VI.

### PROBLEM XVI.

*In a given circle, whoſe diameter is* AB, *to inſcribe a regular octagon.* Fig. 22.

SOLUTION. OPEN the legs of the ſector, till the tranſverſe diſtance of 6 and 6, be equal to AB: Then will the tranſverſe diſtance of 8 and 8, be the ſide of an octagon which will be inſcribed in the given circle.

IN like manner may any other polygon not exceeding 12 ſides, be inſcribed in a given circle.

### PROBLEM XVII.

*On a given line* AB, *to deſcribe a regular pentagon.* Fig. 23.

SOLUTION. 1ſt. Make AB a tranſverſe diſtance to 5 and 5.

2d. AT that opening of the ſector, take the tranſverſe diſtance of 6 and 6; and with this radius, on the points A, B, as centers, deſcribe arcs cutting in C.

3d. ON C as a center, with the ſame radius, deſcribe a circumference paſſing through the points A, B; and in this circle may the pentagon, whoſe ſide is AB, be inſcribed.

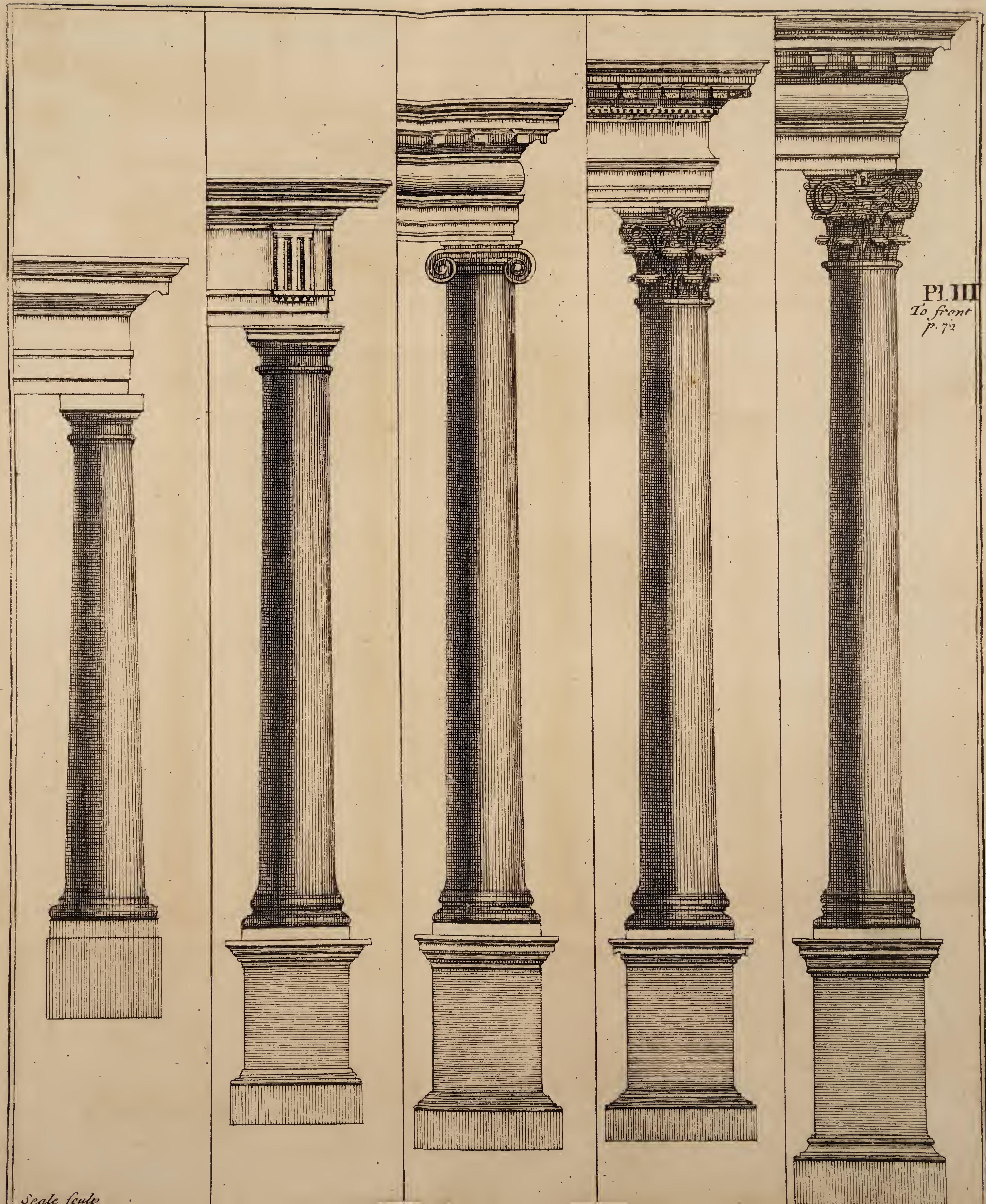
BY a like proceſs may any other polygon, of not more than 12 ſides, be deſcribed on a given line.

THE ſcales of chords will ſolve theſe two problems, or any other of the like kind: Thus,

*In a circle whoſe diameter is* AB, *to deſcribe a regular polygon of* 24 *ſides.* Fig. 24.

SOLUTION. 1ſt. Make the diameter AB, a tranſverſe diſtance to 60 and 60, on the ſcales of chords.

Pl. III
*To front p. 72*



*Scale ſculp*

2d. Divide 360 by 24; the quotient gives 15.

3d. Take the tranſverſe diſtance of 15 and 15, and this will be the chord of the 24th part of the circumference.

As there are great difficulties attending the taking of diviſions accurately from ſcales; therefore in this problem, where a diſtance is to be repeated ſeveral times, it will be beſt to proceed thus.

With the chord of 60 degrees, divide the circumference into ſix equal parts.

In every diviſion of 60 degrees, lay down, 1ſt. The chord of 15 degrees. 2d. The chord of 30 degrees. 3d. The chord of 45 degrees, beginning always at the ſame point.

If methods like this be purſued in all ſimilar caſes, the error in taking diſtances, will not be multiplied into any of the diviſions following the firſt.

## SECT. XIV.

### *Some Uſes of the Scales of Chords.*

THESE double ſcales of chords, are more convenient than the ſingle ſcales, ſuch as deſcribed on the plain ſcale; for on the ſector, the radius with which the arc is to be deſcribed, may be of any length between the tranſverſe diſtance of 60 and 60, when the legs are cloſe, and that of the tranſverſe diſtance of 60 and 60, when the legs are opened as far as the inſtrument will admit of. But with the chords on the plain ſcale, the arc deſcribed, muſt be always of the ſame radius.

## PROBLEM XVIII.

*To protract, or lay down, a right lined angle,* BAC, *which ſhall contain a given number of degrees,* Pl. VI.

Case I. *When the degrees given are under* 60 : *Suppoſe* 46. Fig. 25.

1ſt. At any opening of the ſector, take the tranſverſe diſtance of 60 and 60, (on the chords;) and with this opening, deſcribe an arc BC.

2d. Take the tranſverſe diſtance of the given degrees 46, and lay this diſtance on the arc from any point B, to C ; marking the extremities B, C, of the ſaid diſtance.

3d. From the center A of the arc, draw two lines AC, AB, each paſſing through one extremity of the diſtance BC, laid on the arc ; and theſe two lines will contain the angle required.

Case II. *When the degrees given are more than* 60 : *Suppoſe* 148.

1ſt. Describe the arc BC as before.

2d. Take the tranſverſe diſtance of $\frac{1}{2}$ or $\frac{1}{3}$, of the given degrees 148 ; ſuppoſe $\frac{1}{3} = 49\frac{1}{3}$ degrees ; lay this diſtance on the arc thrice ; *viz.* from B to *a*, from *a* to *b*, from *b* to D.

3d From the center A, draw two lines AB, AD ; and the angle BAD will contain the degrees required.

*When an angle containing leſs than* 5 *degrees, ſuppoſe* $3\frac{1}{2}$, *is to be made, it is moſt convenient to proceed thus.*

1ſt. Describe the arch DG with the chord of 60 degrees.

2d. From ſome point D, lay the chord of 60 degrees to G ; and the chord of $56\frac{1}{2}$ degrees ($= 60^\circ - 3\frac{1}{2}^\circ$) from D to E.

3d. Lines drawn from the center A, through G and E, will form the angle AGE, of $3\frac{1}{2}$ degrees.

If the radius of the arc or circle is to be of a given length; then make the tranſverſe diſtance of 60 and 60, equal to that aſſigned length.

Either of theſe ſcales of chords, may be uſed ſingly in the manner directed in the uſe of chords on the plane ſcale.

From what has been ſaid about the protracting of an angle to contain a given number of degrees, it will be eaſy to ſee how to find the degrees which are contained in a given angle already laid down.

## PROBLEM XIX.

*To delineate the viſual lines of a ſurvey; by having given, the bearings and diſtances from each other, of the ſtations terminating thoſe viſual lines.*

Exam. Suppoſe in the field-book of a ſurvey, the bearings and diſtances of the ſtations were expreſſed as follows:

⊙ ſignifies Station.
B ——— Bearing.
D ——— Diſtance.

| | | | | |
|---|---|---|---|---|
| | ⊙ 1. | B 70°50′ | D 1080 links. | |
| | ⊙ 2. | B 128 10 | D 580. | |
| | ⊙ 3. | B 32 15 | D 605. | |
| | ⊙ 4. | B 287 30 | D 766. | |
| | ⊙ 5. | B 50 45 | D 940. | |
| | ⊙ 6. | B 273 55 | D 1065. | |
| | ⊙ 7. | B 183 25 | D 700. | |
| Return to D 314 in ⊙ 7. | ⊙ 8. | B 133 30 | D 510 | to ⊙ 5. |
| | ⊙ 9. | B 186 30 | D 390 | to ⊙ 2. |
| Return to D 700 in ⊙ 7. | ⊙ 10. | B 209 20 | D 068 | cutting ſt D. |
| Return to ⊙ 10. | ⊙ 11. | B 275 30 | D 800 | |
| | ⊙ 12. | B 171 50 | D 784 | to ⊙ 1. |

The bearings are counted from the North, Eaſtward. Therefore all the bearings under 90 degrees, fall between the N. and E. or in the 1ſt quadrant.

Bearings between 90° and 180°, fall between the E. and S. or in the 2d quadrant.

Those between 180° and 270°, fall between the S. and W. or in the 3d quadrant.

And thoſe between 270° and 360°, fall between the W. and N. or in the 4th quadrant.

Solution. 1ſt. Take from the chords the tranſverſe diſtance of 60 and 60, (the ſector being opened at pleaſure,) with this radius deſcribe a circumference, and draw the diameters NS. WE. at right angles. Pl. VI. Fig. 31.

2d. The firſt bearing 70°. 50′ is in the firſt quadrant, but being more than 60°, take the tranſverſe diſtance of the half of 70° 50′, and apply this extent in the circumference twice from N. towards E, and the point correſponding to the 1ſt bearing will be obtained, which mark with the figure 1.

3d The ſecond bearing 128° 10′, falls in the ſecond quadrant; its ſupplement to 180° is 51° 50′, that is 51° 50′ from the S. point. Now take the tranſverſe diſtance of 51° 50′, and apply it in the circumference from S. towards E, and the point correſponding to the ſecond bearing will be found, which mark with the figure 2.

4th. The 3d bearing 32° 15′, is to be applied from N. to 3: The 4th bearing 287° 30′, is in the 4th quadrant; therefore take it from 360°, and the remainder 72° 30′, is to be applied from N. towards W. and the point 4 repreſenting the 4th bearing will be known.

In this manner proceed with all the other bearings, and mark the correſponding points in the circumference with the numbers 5, 6, 7, *&c.* agreeable to the number of the bearing or ſtation.

5th. Chuſe ſome convenient point on the paper to begin at, as at the place markt ⊙ 1. Lay a parallel ruler by c the centre of the circle, and the point in its circumference marked 1, and (by the help of the ruler) draw a parallel line thro' ⊙ 1, the point choſe for the firſt ſtation, in the direction of the (ſuppoſed) radius C 1; and on this line lay the firſt diſtance; that is, take from a convenient ſized ſcale of equal parts the extent of 1080, and transfer this extent from ⊙ 1 to ⊙ 2; and this line will repreſent the firſt diſtance meaſured, laid down according to its true poſition in reſpect to the circle firſt deſcribed.

6th. Lay the ruler by the centre C, and the point in the circumference noted by the figure 2, and parallel to this poſition of the ruler, draw thro' the point ⊙ 2 a line ⊙ 2 ⊙ 3, in the direction of the (ſuppoſed) radius C 2, and on this line lay from ⊙ 2 to ⊙ 3 the extent 580 taken from the ſame ſcale of equal parts the 1080 was taken from, and this line ſhall repreſent the ſecond meaſured diſtance laid down in its true poſition relative to the firſt diſtance.

Proceed in this manner from ſtation to ſtation until the line ⊙ 7 ⊙ 10 is drawn.

7th. Take from the ſcale of equal parts 314, and apply this extent in the line ⊙ 7 ⊙ 10 from ⊙ 7 to ⊙ 8, and the relative point, where the eighth ſtation was taken, will be repreſented by the point ⊙ 8; then by the parallel ruler draw the line ⊙ 8 ⊙ 5, in the direction of, and parallel to, the (ſuppoſed) radius C 8; and if the preceding work is accurately performed, this line will not only paſs thro' the point ⊙ 5, but the length of the line ⊙ 8 ⊙ 5 will be equal to 510, as the ſtation line was meaſured in the field.

8th. Now as the 9th ſtation falls on the ſame point as the 5th ſtation did, draw the line ⊙ 9 ⊙ 2, and this line will not only be parallel to the (ſuppoſed) radius C 9, but will alſo meaſure on the ſcale of equal parts 390, the length meaſured in the field from the 9th ſtation.

9th. The 10th ſtation is taken from the end of the line 700 meaſured from the 7th ſtation; therefore drawing from ⊙ 10 a line parallel to the (ſuppoſed) radius C 10, this line will concur with the firſt meaſured line at the diſtance of 668 from the point ⊙ 10.

10th Returning to ⊙ 10 again, the ſame point is taken for the 11th ſtation, and the line ⊙ 11 ⊙ 12 is to be drawn parallel to the (ſuppoſed) radius C 11, and to be made of the length of 800 from the ſcale of equal parts; and this will give the point ⊙ 12 for the 12th ſtation: Then drawing the line ⊙ 12 ⊙ 1, if the operation is every where truly done, this line will not only be parallel to the (ſuppoſed) radius C 12, but will alſo meaſure on the ſcale of equal parts 784, the ſame as was meaſured in the field in proceeding from ⊙ 12 to ⊙ 1.

By ſuch methods as theſe, the ſurveyor obtains a cheque on his work, and can make his *ſurvey cloſe* (as 'tis called) as he proceeds.

The drawing of the viſual lines of a ſurvey is, tho' an eſſential part, but a ſmall ſtep towards the making a plan; for the remaining work the reader is refer'd to the treatiſes already extant on that ſubject.

What has been ſaid about the delineating of the viſual lines of a ſurvey, may be applied to navigation in the conſtruction of a figure to repreſent the various courſes and diſtances a ſhip has ſailed in a given time, called traverſe ſailing; for the courſes are the bearings from the Meridian, and the diſtances ſailed are of the ſame kind as the diſtance between ſtation and ſtation in a ſurvey.

## SECT. XV.

### Some Uſes of the Logarithmic Scale of Numbers.

BEFORE any operations can be performed by this ſcale, the notation, or the eſtimating of the values of the ſeveral diviſions, muſt be well known.

Therefore, the Sector being quite opened,

| | | | | | |
|---|---|---|---|---|---|
| If the 1 at the beginning of the ſcale, or of the 1ſt interval, be taken for | 1<br>10<br>100<br>&c<br>$\frac{1}{10}$<br>$\frac{1}{100}$<br>&c. | Then the 1 in the middle, or at the end of the 1ſt interval and the beginning of the ſecond, will expreſs | 10<br>100<br>1000<br>&c.<br>1<br>$\frac{1}{10}$<br>&c. | And the 10 at the end of the 2d interval, or end of the ſcale, will repreſent | 100<br>1000<br>10000<br>&c.<br>10<br>1<br>&c. |

And the primary and intermediate diviſions in each interval, muſt be eſtimated according to the values ſet on their extremities, *viz*, at the beginning, middle and end of the ſcale.

In arithmetical multiplication, or diviſion; the parts may be conſidered as proportional terms; for in ſimple multiplication; as unity or 1, is to one factor; ſo is the other factor, to the product: And in diviſion; as the diviſor, is to unity; (or to the dividend,) ſo is the dividend, (or unity,) to the quotient.

Now as the common logarithms of numbers, expreſs how far the ratios of their correſponding numbers are diſtant from unity; it follows, that of thoſe numbers which are proportional, that is, have equal ratios; their correſponding logarithms will have equal intervals, or diſtances: and hence ariſes the rule for working proportionals on the logarithmic ſcale.

Rule. Set one foot of the compaſſes on the point or diviſion repreſenting the firſt term, and extend the other foot to the point repreſenting the ſecond term: Keep the compaſſes thus opened; ſet one foot on the point expreſſing the third term, and the other foot will fall on the fourth term, or number ſought.

Exam. I. *What is the product of* 3 *by* 4?

Solution. Set one foot on the 1 at the beginning, and extend the other to 3, in the firſt interval; with this opening, ſet one foot on 4, in the firſt interval, and the other foot will reach to 12, found in the ſecond interval.

*Obſerve.* In this Exam. the 1, 3, and 4, are valued as units in the firſt interval; and the one in the middle is 10; the diſtance between this 1 or 10, and the 2 or 20, in the ſecond interval, is divided into 10 principal parts, expreſs'd by the longer ſtrokes; every one in this Exam. is taken as an unit; now as the point of the compaſſes falls on the ſecond of theſe principal parts, that is on 2 units beyond 10; therefore this point is to be eſteemed in this Exam. as 12.

Exam. II. *What is the product of* 40 *by* 3?

Solution. In the firſt interval, take the diſtance between 1 and 3; and this diſtance will reach from (4 or) 40 in the firſt interval to (12 or) 120 in the ſecond interval.

*Obſerve.* The 1 and 3 in the firſt interval, are taken as units: but as the values given to the diviſions in either interval, may as well be call'd 40, as 4; and being taken as 40, the 1 at the beginning of the ſecond interval will be 100; and the 2 in the ſecond interval will be 200: conſequently the principal diviſions between this 1 and 2 will each expreſs 10; and ſo the ſecond of them will be 20, which with the 100, expreſs'd by the 1, makes 120.

Exam. III. *What is the product of* 35 *by* 24?

Solution. The diſtance from 1 in the firſt interval, to 24 in the ſecond, will reach from 25 in the firſt interval, to 840 in the ſecond.

*Obſerve.* In the firſt application of the compaſſes, the primary diviſions in the firſt interval are taken as units, and thoſe in the ſecond interval, as tens: But in the ſecond application, the primary diviſions in the firſt interval are reckon'd as tens; and thoſe in the ſecond, as hundreds.

As the extent out of one interval into the other, may ſometimes be inconvenient, it will be proper to ſee in ſuch caſes, how the example may be ſolved in one interval. Thus,

In either interval, take the extent from 1 to $2\frac{4}{10}$ (i. e. 24) and this extent, (in either interval) will reach from $3\frac{5}{10}$ (i. e. 35) to $8\frac{4,0}{10,0}$; (i. e. 840.)

In this operation; the ſecond term is reckoned a tenth higher than the firſt term; therefore, as it falls in the ſame interval, the fourth term muſt be a tenth higher than the third term.

Exam. IV. *What is the product of* 375 *by* 60?

Solution. The extent from 1 to 6, (or 60) in the firſt interval will reach from $3\frac{7\frac{1}{2}}{10}$ (=$3\frac{75}{100}$ or 375) in the firſt interval, to $2\frac{25}{100}$ in the ſecond interval; which diviſion muſt be reckoned 22500: For had the point fell in the firſt interval, it would have been one place more than the 375, becauſe 60 is one place more than 1; but as it falls in the ſecond interval, every of whoſe diviſions is one place higher than thoſe in the firſt interval, therefore, it muſt have two places more than 375, which is taken in the firſt interval.

If the operations in theſe examples be well conſidered, it will not be difficult to apply others to the ſcale, and readily to aſſign the value of the reſult.

Exam. V. *What will be the quotient of* 36 *divided by* 4?

Solution. The extent from 4 to 1, in the firſt interval, will reach from 36 in the ſecond interval to nine in the firſt.

It is to be obſerved, that when the ſecond term is greater than the firſt term; the extents are reckoned from the left hand towards the right: and when the ſecond term is leſs than the firſt, the extents are taken from the right hand towards the left: that is, the extents are always counted the ſame way towards which the terms proceed.

EXAM. VI. *If* 144 *be divided by* 9; *what will be the quotient?*

SOLUTION. The extent from 9 to 1, will reach from 144 to 36.

EXAM. VII. *If* 1728 *be divided by* 12; *what will be the quotient?*

SOLUTION. The extent from 12 to 1, will reach from 1728 to 144.

EXAM. VIII. *To the numbers* 3, 8, 15; *find a fourth proportional.*

SOLUTION. The extent from 3 to 8; will reach from 15 to 40.

EXAM. IX. *To the numbers* 5, 12, 38; *find a 4th proportional.*

SOLUTION. The extent from 5 to 12, will reach from 38 to 91 $\frac{1}{5}$.

EXAM. X. *To the numbers* 18, 4, 364; *find a 4th proportional.*

SOLUTION. The extent from 18 to 4; will reach from 364 to 80 $\frac{8}{9}$.

EXAM. XI. *To two Numbers* 1 *and* 2; *to find a ſeries of continued proportionals.*

SOLUTION. The extent from 1 to 2, will reach from 2 to 4; from 4 to 8 in the firſt interval; from 8 to 16 in the ſecond interval; from 16 to 32; from 32 to 64; *&c.* Alſo the ſame extent will reach from 1 $\frac{1}{2}$ to 3; from 3 to 6; from 6 to 12; from 12 to 24; from 24 to 48; *&c.* And the ſame extent will reach from 2 $\frac{1}{2}$ to 5; from 5 to 10; from 10 to 20; from 20 to 40; *&c.* And in a like manner proceed, if any other ratio was given beſides that of 1 to 2.

This Example is of uſe, to find if the diviſions of the line of numbers, are accurately laid down on the ſcale.

There are many other uſes to which this ſcale of log. numbers are applicable, and on which ſeveral large treatiſes have been wrote; but the deſign here, is not to enter into all the uſes of the ſcales on the ſector, only to give a few examples thereof: but after all that has been ſaid, when examples are to be wrought whoſe reſult exceeds three places, 'tis beſt to do it by the pen, for on inſtruments, altho' they be very large ones, the loweſt places of the anſwers, at beſt, are but gueſs'd at.

## SECT. XVI.

## *Some uſes of the Scales of Log. Sines and Log. Tangents.*

THESE ſcales are chiefly uſed in the ſolution of the caſes of plain and ſpherical trigonometry, which will be fully exemplified hereafter: But in this place, it will be proper to ſhew, how proportional terms are applied to the ſcales.

In plane trigonometrical proportions, there are always four terms under conſideration; ſuppoſe two ſides and two angles, whereof, only three of the terms are given, and the fourth is required: Now the ſides in plane trigonometry, are always applied to the ſcale of log. numbers; and the angles are either applied to the log. ſines, or to the log. tangents; according as the ſines or tangents are concerned in the proportion. Therefore, when among the three things given, if two of them be ſides, and the other an angle; or if two terms be angles, and the other a ſide.

RULE. On the ſcale of log. numbers, take the extent between the diviſions expreſſing the ſides; and this extent applied from the diviſion expreſſing the angle given, will reach to the diviſion ſhewing the angle required.

OR, the extent of the angles taken, will reach from the ſide given to the ſide required, on the line of numbers.

So in ſpherical trigonometry, where ſome of the caſes are worked wholly on the ſines, others partly on ſines, and partly on tangents; the extent taken with the compaſſes, between the firſt and ſecond terms, when thoſe terms are of the ſame kind, will reach from the third term to the fourth.

OR, the extent from the firſt term to the third, when they are of the ſame kind, will reach from the ſecond term to the fourth.

# SECT. XVII.

## *Some uſes of the double Scales of Sines, Tangents, and Secants.*

## PROBLEM XX.

*Given the radius of a circle (ſuppoſe equal to* 2 *inches) required the ſine, and tangent of* 28° 30′ *to that radius.*

SOLUTION. Open the ſector ſo that the tranſverſe diſtance of 90 and 90, on the ſines; or of 45 and 45 on the tangents; may be equal to the given radius; *viz.* two inches: Then will the tranſverſe diſtance of 28° 30′, taken from the ſines, be the length of that ſine to the given radius; or if taken from the tangents, will be the length of that tangent to the given radius.

*But if the ſecant of* 28°·30′ *was required?*

MAKE the given radius two inches, a tranſverſe diſtance to 0 and 0, at the beginning, of the line of ſecants; and then take the tranſverſe diſtance of the degrees wanted, *viz.* 28° 30′.

*A Tangent greater than* 45 *degrees* (*ſuppoſe* 60 *degrees*) *is found thus.*

MAKE the given radius, ſuppoſe 2 inches, a tranſverſe diſtance to 45 and 45 at the beginning of the ſcale of upper tangents; and then the required degrees 60° 00′ may be taken from this ſcale.

The ſcales of upper tangents and ſecants do not run quite to 76 degrees; and as the tangent and ſecant may be ſometimes wanted to a greater number of degrees than can be introduced on the ſector, they may be readily found by the help of the annexed table of the natural tangents and ſecants of the degrees above 75; the radius of the circle being unity.

| Degrees. | Nat. Tangent. | Nat. Secant. |
|---|---|---|
| 76 | 4,011 | 4,133 |
| 77 | 4,331 | 4,445 |
| 78 | 4,701 | 4,810 |
| 79 | 5,144 | 5,241 |
| 80 | 5,671 | 5,759 |
| 81 | 6,314 | 6,392 |
| 82 | 7,115 | 7,185 |
| 83 | 8,144 | 8,205 |
| 84 | 9,514 | 9,567 |
| 85 | 11,430 | 11,474 |
| 86 | 14,301 | 14,335 |
| 87 | 19,081 | 19,107 |
| 88 | 28,636 | 28,654 |
| 89 | 57,290 | 57,300 |

Meaſure the radius of the circle uſed, upon any ſcale of equal parts. Multiply the tabular number by the parts in the radius, and the product will give the length of the tangent or ſecant ſought, to be taken from the ſame ſcale of equal parts.

Exam. *Required the length of the tangent and ſecant of* 80 *degrees to a circle whoſe radius, meaſured on a ſcale of* 25 *parts to an inch, is* $47\frac{1}{2}$ *of thoſe parts?*

| | tangent. | ſecant. |
|---|---|---|
| Againſt 80 degrees ſtands | 5,671 | 5,759 |
| The radius is | 47,5 | 47,5 |
| | 28355 | 28795 |
| | 39697 | 40313 |
| | 22684 | 23036 |
| | 269,3725 | 273,5525 |

So the length of the tangent on the twenty-fifth ſcale will be $269\frac{1}{3}$ nearly. And that of the ſecant about $273\frac{1}{2}$.

Or thus. The tangent of any number of degrees may be taken from the ſector at once; if the radius of the circle can be made a tranſverſe diſtance to the complement of thoſe degrees on the lower tangent.

Exam. *To find the tangent of* 78 *degrees to a radius of* 2 *inches.*

Make two inches a tranſverſe diſtance to 12 degrees on the lower tangents; then the tranſverſe diſtance of 45 degrees will be the tangent of 78 degrees.

In like manner the ſecant of any number of degrees may be taken from the ſines, if the radius of the circle can be made a tranſverſe diſtance to the coſine of thoſe degrees. Thus making two inches a tranſverſe diſtance to the ſine of 12 degrees; then the tranſverſe diſtance of 90 and 90 will be the ſecant of 78 degrees.

From hence it will be eaſy to find the degrees anſwering to a given line, expreſſing the length of a tangent or ſecant, which is too long to be meaſured on thoſe ſcales, when the ſector is ſet to the given radius.

Thus, For a tangent, make the given line a tranſverſe diſtance to 45 and 45 on the lower tangents; then take the given radius and apply it to the lower tangents; and the degrees where it becomes a tranſverſe diſtance is the cotangent of the degrees anſwering to the given line.

And for a ſecant. Make the given line a tranſverſe diſtance to 90 and 90 on the ſines. Then the degrees anſwering to the given radius applied as a tranſverſe diſtance on the ſines, will be the co-ſine of the degrees anſwering to the given ſecant line.

## PROBLEM XXI.

*Given the length of the ſine, tangent, or ſecant, of any degrees; to find the length of the radius to that ſine, tangent, or ſecant.*

Make the given length, a tranſverſe diſtance to its given degrees on its reſpective ſcale: Then,

*In the ſines.* The tranſverſe diſtance of 90 and 90 will be the radius ſought.

*In the lower tangents.* The tranſverſe diſtance of 45 and 45 near the end, of the ſector will be the radius ſought.

*In the upper tangents.* The tranſverſe diſtance of 45 and 45 taken towards the centre of the ſector on the line of upper tangents, will be the centre ſought.

*In the ſecants.* The tranſverſe diſtance of 0 and 0, or the beginning of the ſecants, near the centre of the ſector, will be the radius ſought.

## PROBLEM XXII.

*Given the radius and any line repreſenting a ſine, tangent or ſecant; to find the degrees correſponding to that line.*

Solution. Set the ſector to the given radius, according as a ſine, or tangent, or ſecant is concerned.

Take the given line between the compaſſes; apply the two feet tranſverſly to the ſcale concerned, and ſlide the feet along till they both reſt on like diviſions on both legs; then will thoſe diviſions ſhew the degrees and parts correſponding to the given line.

## PROBLEM XXIII.

*To find the length of a verſed ſine to a given number of degrees, and a given radius.*

Make the tranſverſe diſtance of 90 and 90 on the ſines, equal to the given radius.

Take the tranſverſe diſtance of the ſine complement of the given degrees.

If the given degrees are leſs than 90, the difference between the ſine complement and the radius, gives the verſed ſine.

If the given degrees are more than 90, the ſum of the ſine complement and the radius, gives the verſed ſine.

## PROBLEM XXIV.

*To open the legs of the ſector, ſo that the correſponding double ſcales of lines, chords, ſines, tangents, may make, each, a right angle.*

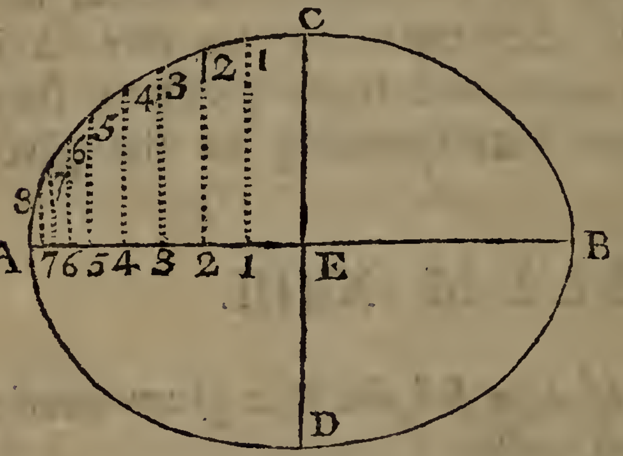
*On the lines*, make the lateral diſtance 10, a diſtance between 8 on one leg, and 6 on the other leg.

*On the ſines.* make the lateral diſtance 90, a tranſverſe diſtance from 45 to 45; or from 40 to 50; or from 30 to 60; or from the ſine of any degrees, to their complement.

*Or on the ſines*, make the lateral diſtance of 45 a tranſverſe diſtance between 30 and 30.

## PROBLEM XXV.

*To deſcribe an Ellipſis, having given* AB *equal to the longeſt diameter*; *and* CD *equal to the ſhorteſt diameter:*



Solution. 1ſt. Set the two diameters AB, CD, at right angles to each other in their middles at E.

2d. Make AE a tranſverſe diameter to 90 and 90 on the ſines; and take the tranſverſe diſtances of 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, ſucceſſively, and apply thoſe diſtances to AE from E towards A, as at the points 1, 2, 3, 4, 5, 6, 7, 8; and thro' thoſe points draw lines parallel to EC.

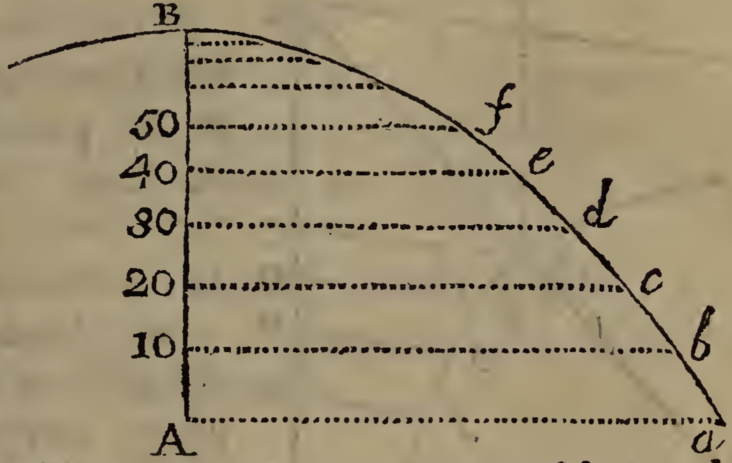
3d. Make EC a tranſverſe diſtance to 90 and 90 on the ſines; take the tranſverſe diſtances of 80°, 70°, 60°, 50°, 40°, 30°, 20°, 10°, ſucceſſively, and apply thoſe diſtances to the parallel lines from 1 to 1, 2 to 2, 3 to 3, 4 to 4, 5 to 5, 6 to 6, 7 to 7, 8 to 8, and ſo many points will be obtained thro' which the curve of the ellipſis is to paſs,

The ſame work being done in all the four quadrants, the elliptical curve may be compleated.

This Problem is of conſiderable uſe in the conſtruction of ſolar Eclipſes; but inſtead of uſing the ſines to every ten degrees, the ſines belonging to the degrees and minutes correſponding to the hours, and quarter hours are to be uſed.

## PROBLEM XXVI.

*To deſcribe a Parabola whoſe parameter ſhall be equal to a given line.*



SOLUTION 1ſt. Draw a line to repreſent the axis, in which make AB equal to half the given parameter; divide AB like a line of ſines to every ten degrees, as at the points 10, 20, 30, 40, 50, *&c.* and thro' theſe points draw lines at right angles to the axis AB.

2d. MAKE the lines A*a*, 10*b*, 20*c*, 30*d*, 40*e*, *&c.* reſpectively equal to the chords of 90° 80°, 70°, 60°, 50°, *&c.* to the radius AB, and the points *a*, *b*, *c*, *d*, *e*, *&c.* will be in the curve of a parabola.

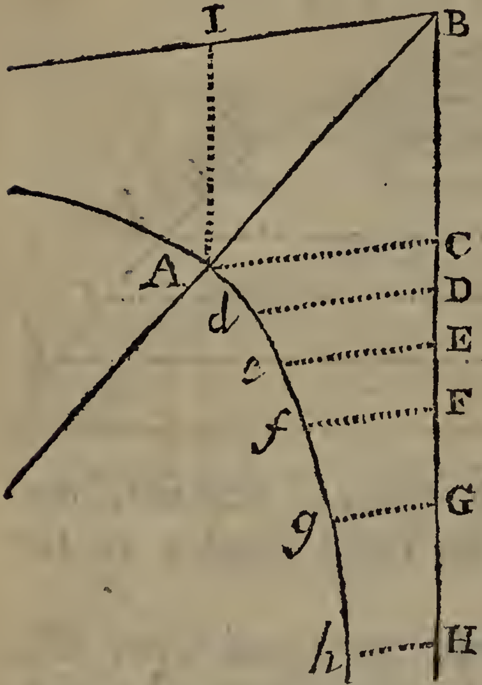
Therefore a ſmooth curve line drawn thro' thoſe points and the vertex B, will repreſent the parabolic curve required.

The like work may be done on both ſides of the axis when the whole curve is wanted.

As the chords on the ſector run no farther than 60, thoſe of 70, 80 and 90 may be found by taking the tranſverſe diſtance of the ſines of 35°, 40°, 45° to the radius AB, and applying thoſe diſtances twice along the lines 20*c*, 10*b*, *&c.*

# PROBLEM XXVII.

*To deſcribe an Hyperbola, the vertex* A *and aſſymptotes* BH, BI, *being given.*



SOLUTION 1ſt. The aſſymptotes BH, BI, being drawn in any poſition, the line BA, biſecting the angle IBH, and the vertex A taken, draw AI, AC, parallel to BH, BI.

2d. Make AC a tranſverſe diſtance to 45 and 45 on the upper tangents, and apply to the aſſymptotes from B, ſo many of the upper tangents taken tranſverſly as may be thought convenient, as BD 50°, BE 55°, BF 60°, BG 65°, BH 70°, *&c.* and draw D*d*, E*e*, *&c.* parallel to AC.

3d. Make AC a tranſverſe diſtance to 45 and 45 on the lower tangents, take the tranſverſe diſtance of the co-tangents before uſed, and lay them on thoſe parallel lines; thus make D*d*=40°, E*e*=35°, Ff=30°, G*g*=25°, H*h*=20°, *&c.* and thro' the points A, *d*, *e*, *f*, *g*, *h*, *&c.* If a curve line be drawn it will be the hyperbola required.

There are many other methods of conſtructing the curves in the three laſt problems, and a multitude of entertaining and uſeful properties which ſubſiſt among the lines drawn within and about theſe curves, which the inquiſitive reader will find in the treatiſes on conic ſections.

## PROBLEM XXVIII.

*To find the diſtance of places on the terreſtrial globe by having given their latitudes and longitudes.*

This problem conſiſts of ſix caſes.

Case I. If both the places are under the equator.

Then the difference of longitude is their diſtance.

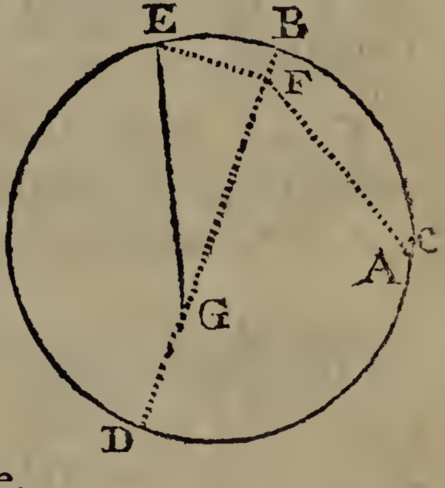
Case II. When both places are under the ſame meridian.

Then the difference of latitude is their diſtance.

Case III. When only one of the places has latitude, but both have different longitudes.

Exam. *Iſland of* Bermudas, *lat.* 32° 25′ N. *longit.* 68° 38′ W. *Iſland of* St. Thomas, *lat* 0 0, *longit.* 1° 0 E.

Required their diſtance.



Solution 1ſt. With the chord of 60° deſcribe a circle repreſenting the equator, wherein take a point c to repreſent the beginning of longitude.

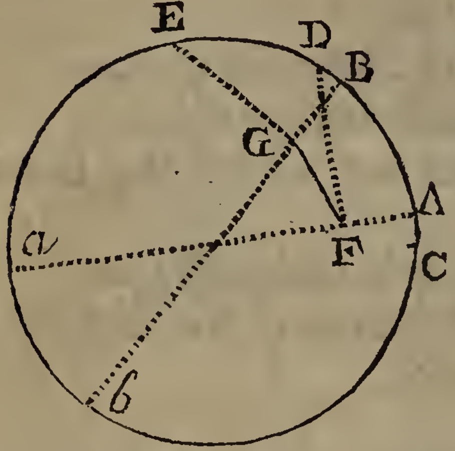
2d. From c apply the chord of *Bermudas* longitude 68° 38′ to b, and that of *St. Thomas*'s longitude to a, the arc ab, being the difference of longitude.

3d. From b, the place having latitude, draw the diameter bd, apply the chord of the latitude 32° 25, from b to e, and draw ef at right angles to bd.

4th. Draw fc, make fg, equal to fc, and draw eg; then eg meaſured on the chords will give the diſtance ſought, about 73 degrees.

Case IV. When the given places are in the ſame parallel of latitude.

EXAM. *Required the diſtance between the* Lizard *and* Pengwin *Iſland, both in latitude* 49°, 56′ N. *the longitude of the* Lizard *being* 5° 14′ W. *and that of* Pengwin *Iſland* 50° 32′ W.

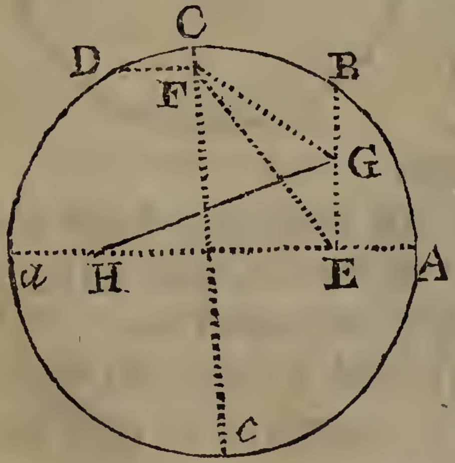


SOLUTION 1ſt. From C, the commencement of the longitude, apply the chord of the *Lizard*'s longitude to A, and of *Penguin*'s longitude to B, and draw the diameters A*a*, B*b*.

Apply the chord of the common latitude 49° 56′ from A to D, and from B to E; draw DF and EG at right angles to A*a*, B*b*, and join GF; then GF meaſured on the chords will give the diſtance ſought, about 29 degrees.

CASE V. When the given places are on the ſame ſide of the equator, but differ both in latitude and longitude.

EXAM. *What is the diſtance between* London *in latitude* 51° 32′ N. *longitude*, 0° 0′ *and* Bengal *in latitude* 22° 0′ N. *longitude* 92° 45′ E.



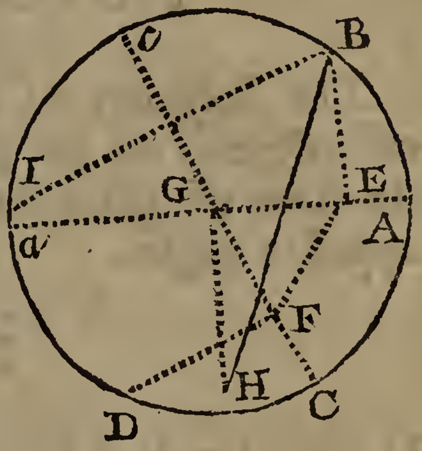
SOLUTION. From A, *London*'s longitude, apply *Bengal*'s longitude 92° 45′ to C, taken from the chords; alſo apply the chord of *London*'s latitude from A to B, and of *Bengal*'s latitude from C to D.

2d. Draw the diameters A*a*, C*c*, and BE, DF, at right angles to A*a*, C*c*, and join FE.

3d. Make BG equal to DF, and EH equal to EF, join GH; Thus GH meaſured on the chords will give the diſtance required, which is about 72 degrees.

CASE VI. When the places are on contrary ſides of the equator, and differ both in latitude and longitude.

EXAM. *What is the diſtance between* London *in latitude* 51° 32′ N. *longitude* 0° 0′ *and* Cape-Horn *in latitude* 55° 42′ S. *longitude* 66° 00′ W.



SOLUTION 1ſt. From A, *London*'s longitude, apply the chord of *Cape-Horn*'s longitude to C, draw the diameters A*a*, C*c*; alſo apply the chords of *London*'s latitude from A to B, and of *Horn*'s latitude from C to D.

2d. Draw BE and DF at right angles to A*a*, C*c*; join EF and make EG equal to EF.

3d. At right angles to A*a*, draw GH, and make it equal to DF; join BH, which meaſured on the chords will give the diſtance required, which is about 123 degrees.

*To meaſure* BH *on the chords*; apply BH from B to I, and meaſure the arc B*c* I.

## SECT. XVII.

## *The Uſe of ſome of the ſingle and double Scales, applied in the Solution of the Caſes of plain Trigonometry.*

### PROBLEM XXIX.

IN *any right lin'd plane triangle, any three of the ſix terms,* viz. *ſides and angles, (provided one of them be a ſide) being given, to find the other three.*

This problem conſiſts of three caſes.

CASE I. when among the things given, there be a ſide and its oppoſite angle.

CASE II. When there is given two ſides and the included angle.

CASE III. When the three ſides are given.

### SOLUTION *of* CASE I.

The Solution of the examples falling under this caſe depend on the proportionality there is between the ſides of plane triangles, and the ſines of their oppoſite angles.

### EXAMPLE I.

In the triangle ABC: Given $\left.\begin{array}{l}AB=56\\ AC=64\end{array}\right\}$ equal parts.

$\angle B=46^\circ\ 30'$

Required $\angle C, \angle A$, & BC.

The proportions are as follow,

As ſide AC : ſide AB : : ſine ∠ B : ſine ∠ C.

Then the ſum of the angles B and C being taken from 180°, will leave the angle A.

And as ſine ∠B : ſine ∠A : : ſide AC : ſide CB.

*Firſt by the logarithm ſcales.*

*To find the angle* C.

The extent from 64 (=AC) to 56 (=AB) on the ſcales of logarithm numbers, will reach from 46° 30′ (=∠B) to 39° 24′, (=∠C.) on the ſcale of logarithm ſines.

And the ſum of 46° 30′ and 39° 24′ is 85° 54′

Then 85° 54′ taken from 180°, leaves 94° 6′ for the angle A.

### *To find the ſide* BC.

The extent from 46° 30′ (=∠B) to 85° 54′ the ſupplement of 94° 6′ (=∠A) on the ſcale of log. ſines, will reach from 64 (=AC), to 88 (=BC), on the ſcale of logarithm numbers.

### *Secondly by the double Scales.*

### *To find the Angle* C.

1. Take the lateral diſtance of 64 (=AC) from the lines.
2. Make this a tranſverſe diſtance of 46° 30′ (=∠B) on the ſines.
3. Take the lateral diſtance of 56 (=AB) on the lines.
4. Find the degrees to which this extent is a tranſverſe diſtance on the ſines, *viz.* 39° 24′; and this is the angle ſought.

### *To find the ſide* BC.

1. Take the lateral diſtance of 64 (=AC) from the lines.
2. Make this a tranſverſe diſtance of 46° 30′ (=∠B) on the ſines.
3. Take the tranſverſe diſtance of 85° 54′ (the ſupplement of 94° 6′ = ∠ A) on the ſines.
4. Find the lateral diſtance this extent is equal to, on the lines; and this diſtance, *viz.* 88, will be the ſide required.

Ex. II. In the triangle ABC Pl. VI. Fig. 27.

Given BC = 74
∠B = 104° 0′
∠C = 28 0
Required AB & AC.

Now the ſum of 104° 0′ and 28° 0′ is 132° 0′.

And 132° 0′ taken from 180, leaves 48° 0′ for the angle A.

The proportions are,
As ſine ∠A : ſine ∠C : : ſide BC : ſide AB.
And as ſine ∠A : ſine ∠B : : ſide BC : ſide AC.

### *Firſt, by the Logarithm Scales.*

### *To find* AB.

THE extent from 48° 0′ (= ∠A) to 28° 0′ (=∠C) on the ſcale of logarithm ſines, will reach from 74 (=BC) to 46, 75, (=AB,) on the ſcale of logarithm numbers.

### *To find* AC.

THE extent from 48° 0′ to 76° 0′ (= ſupplement of 104° 0′) on the ſcale of log. ſines, will reach from 74 to 96, 0 (=AC) on the ſcale of logarithm numbers.

### *Secondly by the double Scales.*

### *To find* AB.

1. TAKE the lateral diſtance 74 (= BC) on the lines.

2. MAKE this extent a tranſverſe diſtance to 48° 0′ (= ∠A) on the ſines.

3. TAKE the tranſverſe diſtance of 28° 0′ (=∠C) on the ſines.

4. To this extent find the lateral diſtance on the lines, *viz.* 46,75 and this will be the length of AB.

### *To find* AC.

1. TAKE the lateral diſtance 74 (= BC) on the lines.

2. MAKE this extent a tranſverſe diſtance to 48° 0′ (= ∠A) on the ſines.

3. TAKE the tranſverſe diſtance to 76° 0′ the ſupplement of 104° 0′ (= ∠B) on the ſines.

4. To this extent, find the lateral diſtance on the lines, *viz.* 96, 6, and this will be the length of AC.

## SOLUTION *of* CASE II.

THE ſolution of this caſe depends on a well known theorem, *viz.*

As the ſum of the given ſides
Is to the difference of thoſe ſides,
So is the tangent of the half ſum of the unknown angles
To the tangent of the half difference of thoſe angles.

And the angles are readily found by their half ſum and half difference being known.

Ex. III. In the triangle ABC, Pl. VI. Fig. 28.
Given BC = 74
BA = 52
∠B = 68° 0′
Required ∠A; ∠C; & AC.

### *Preparation.*

TAKE the given angle 68° 0′ from 180°, and half the remainder, *viz.* 56° 0′ is the half ſum of the unknown angles which call z; and let x ſtand for the half difference of thoſe angles.

ALSO find the given ſum of the ſides, *viz.* BC+BA =126.

AND take the difference of thoſe ſides, *viz.* BC—BA=22.

Then the proportions are

As BC+BA : BC—BA : : *tan.* z : *tan.* x.

THEN the ſum of z and x gives the greater angle A.

The difference of z and x gives the leſſer angle c,

And as ſine ∠c : ſine ∠b : : ſine ba : ſide ac.

### *Firſt by the Logarithm Scales.*

### *To find the tangent of* n.

Take the extent from 126 (= ſum of the given ſides) to 22 (= diff. of thoſe ſides) on the ſcale of logarithm numbers; lay this extent from 45° 0′ to the left on the logarithm tangents; ſtay the loweſt point, and bring that which reſted on 45 degrees, to 56° 0′; remove the compaſs, and this extent laid from 45° 0′ towards the left, gives 14 . 31′ equal n.

Then the ſum of 56° 0′ and 14° 31′ or 70° 31′ is the angle a.

And 14° 31′ taken from 56° 0′ leaves 41° 29′ for he angle c.

### *To find* ac.

The extent from 41° 29′ (= ∠c) to 68° 0′ (= ∠ b) on the logarithm ſines, will reach from 52 (= ba) to 72, 75 (= ac) on the ſcale of logarithm numbers.

In finding the tangent of (n, or) the half difference of the unknown angles, there were two applications of the compaſſes to the ſcale of tangents: Now this happens becauſe the upper tangents which ſhould have been continued beyond 45°, or to the right hand, are laid down backwards, or to the left hand, among the lower tangents (the logarithmic tangents aſcending and deſcending by like ſpaces at equal diſtances on both ſides of 45°), and thereby the length of the ſcale is kept within half the length neceſſary to lay down all the tangents in order, from the left towards the right. But ſuppoſing they were ſo laid down, then the point of 56° 0′ will reach as far to the right of 45° as it does now to the left, and the extent on the numbers from 126 to 22 would reach from the point 56° taken on the right of 45°, to 14° 31′ at one application; the ſaid extent being applied from 45° downwards, will reach as far beyond 14° 31′, as is the diſtance from 45° to 56°; therefore the legs of the compaſſes being brought as much cloſer as is that interval, will reach from 45° to the degrees wanted.

INDEED when the half ſum is leſs than 45°, then the extent from the ſum of the ſides to their difference, will reach from the tangent of the half ſum, downward, to the tangent of the half difference, at once.

AND when the half ſum of the unknown angles, and their half difference, are both greater than 45°, then the extent from the ſum of the ſides to their difference, will reach from the tangent of the half ſum of the angles, upwards (or to the right) to the tangent of the half difference of thoſe angles, at once.

### *Secondly by the double Scales.*

Becauſe 126 the ſum of the ſides will be longer than the ſcales of lines, therefore take 63, the half of 126, and 11, the half of 22, the difference of the ſides; for the ratio of 63 to 11, is the ſame as that of 126 to 22. Then

1. TAKE the lateral diſtance 63 on the ſcales of lines.

2. MAKE this extent a tranſverſe diſtance to 56 degrees, on the upper tangents.

3. TAKE the tranſverſe diſtance of 45° on the upper tangents, and make this extent a tranſverſe diſtance to 45° on the other tangents.

4. TAKE the lateral diſtance 11, on the lines.

5. To this extent, find the tranſverſe diſtance on the tangents, and this will be 14° 31′ = N.

AND this is the manner of operation, when M is greater than 45 degrees, and N is leſs.

But when M { greater } than 45
& N are each { leſs } degrees.

Then the third article of the foregoing operation is omitted.

Now having found the angles A and C, the ſide AC may be found as in the firſt or ſecond examples.

But in this caſe, the third ſide AC may be found without knowing the angles. Thus,

1. Take the lateral diſtance of (34 deg.) the half of (68,) the given angle, from the ſines.
2. Make this extent a tranſverſe diſtance, to 30 on the ſines.
3. With the ſector thus opened, take the diſtance from 74 on one leg, to 52 on the other leg, each reckon'd on the lines.
4. The lateral diſtance, on the lines, of this extent, gives the ſide AC = 72, 75.

From the two firſt articles of this operation, is learn'd how to ſet the double ſcales to any given angle.

When the included angle B is 90 degrees, the angles A and C are more readily found, as in the following example, whoſe ſolution depends on this principle. That one of the given ſides has the ſame proportion to radius, as the other given ſide has to the tangent of its oppoſite angle.

Ex. IV. In the triangle ABC: Fig. 29.

Given AB = 45
BC = 65
∠B = 90
Required ∠A ; ∠C ; & AC.

The proportions are,

*For the Angle* A.

As ſide AB : ſide BC : : radius : *tan.* ∠ A.

AND the ∠A taken from 90° leaves the ∠C, then AC may be found as directed in the laſt example.

### *Firſt by the logarithmic Scales.*

THE extent from 45 (= AB) to 65 (= BC) on the numbers, will reach from 45 degrees to 55° 18′ (= ∠A) on the tangents.

HERE the angle A is taken equal to 55° 18′, becauſe the ſecond term BC is greater than the firſt term AB: But if the terms were changed, and it was made BC to AB, then the degrees found would be 34° 42′ = ∠C.

### *Secondly by the double Scales.*

1. TAKE the lateral diſtance of the firſt term, from the lines.
2. MAKE this a tranſverſe diſtance to 45 deg. on the tangents.
3. Take the lateral diſtance of the ſecond term, from the lines.
4. THE tranſverſe diſtance of this extent, found on the tangents, gives the degrees in the angle ſought.

IF the firſt term is greater than the ſecond, then the lateral diſtance of the firſt term, muſt be ſet to 45 degrees on the lower tangents, and the lateral diſtance of the ſecond term, muſt be reckon'd on the ſame tangents.

BUT if the firſt term is leſs than the ſecond, then the lateral extent of the firſt term muſt be ſet to 45° on the upper tangents, and the lateral extent of the ſecond term muſt be reckon'd on the ſame tangents.

### SOLUTION *of* CASE III. Fig. 30.

IN the triangle ABC:

Given BC = 926.
BA = 558.
AC = 702.
Requir'd ∠B, ∠C. ∠A.

There are uſually given for the ſolution of this caſe by the logarithmic ſcales two methods; the one beſt when all the angles are to be found, the other beſt when one angle only is wanted; both methods will be here delivered.

### First. *When all the angles are wanted.*

Suppose a perpendicular AD (Pl. VI. Fig. 30.) drawn to the greateſt ſide BC, from the angle A oppoſite thereto; then AD divides the triangle ABC into two right angled triangles BDA, CDA; in which if CD and DB were known, the angles would be found, as in the ſolution of Caſe I.

Take the ſum of the ſides AC and AB, which is 1260.

Also their difference, which is 144.

Then on the ſcale of numbers, the extent from 926 (= BC) to 1260, will reach from 144 to 196.

And the half ſum of 926 and 196, is 561 = DC.

And the half difference of 926 and 196 is 365 = DB.

The extent from 558 (= BA) to 365 (= BD) on the numbers, will reach on the log. ſines from 90° (= ∠BDA) to 40° 52′ (= ∠BAD.)

Then 40° 52′ taken from 90°, leaves 49° 8′ for ∠B.

And the extent from 702 (= CA) to 561 (= CD) on the numbers, will reach from 90° (= ∠CDA) to 53° 04′ (= ∠CAD) on the ſcale of log. ſines.

Then 53° 4′ taken from 90°, leaves 36° 56′ for the ∠C.

Also the ſum of 40° 52′ and 53° 4′ gives 93° 56′ for the ∠CAB.

SECONDLY, *To find either angle; ſuppoſe* B.

Preparation.

TAKE the difference between BC and BA, the ſides including the angles ſought, and call it D = 368.

FIND the half ſum of AC and D, call it Z = 535

AND the half diff. of AC and D, call it X = 167

THEN as 1 : $\sqrt{\frac{Z \times X}{AB \times BC}}$ : : radius : ſine $\frac{1}{2}$ ∠B.

1. THE extent on the log. numbers from 1 to 535 (= Z), will reach from 167 (= X) to a 4th point; mark it and call it G.

2. THE extent from 1 to 558 (= AB), will reach from 926 (= BC) to a 4th point; mark it and call it H.

3. THE extent from the point H to the point G, will reach from 1, downward to a 4th point, mark it and call it K.

4. THE extent from K, to the middle point between it and the 1 next above K, taken on the log. numbers, will reach on the log. ſines from 90° to 24° 34′, which doubled gives 49° 8′ for the angle B.

BUT the ſcale of log. verſed ſines being uſed, the work will be conſiderably ſhortened. Thus,

1. ON the log. numbers take the extent from 535 (=Z) to 926 (= BC), this will reach from 558 (=BA) to a 4th point, where let the foot of the compaſſes reſt.

2. THEN the extent from that 4th point to 167 (=X), will reach on the line of verſed ſines from 0 degrees (at the end) to 130° 52′, which taken from 180° leaves 49° 8′ for the angle B.

*By*

### *By the double, or ſectoral, Scales.*

### *To find the angle* B.

1. TAKE the lateral diſtance 702 (= AC, the ſide oppoſite to the angle B) from the lines.

2. OPEN the legs of the ſector until this extent will reach from 926 (= CB) on one ſcale of lines, to 558 (= AB) on the other ſcale of lines.

3. THE ſector being thus opened, take the tranſverſe diſtance between 30° and 30° on the ſines, this diſtance meaſured laterally on the ſines, one foot being on the centre, will give 24° 34′ for half the angle B.

THE other angles may be found as ∠B was, or according to the directions in ſome of the preceding caſes.

ALTHOUGH in theſe examples, oblique triangles were taken as being the moſt general; yet it may be readily ſeen, that thoſe concerning right-angled triangles are only particular caſes, and may be, for the general, more eaſily ſolved.

VARIETY of other examples, ſhewing the uſes of theſe ſcales, might be given in various parts of the mathematics, which the reader may of himſelf ſupply: However here will be ſubjoined a few in ſpherical trigonometry, as they will include ſome operations not only curious, but perhaps not to be met with elſewhere.

## SECT. XVIII.

### *The Conſtruction of the ſeveral caſes of Spherical Triangles by the Scales on the Sector.*

THE caſes of ſpherical triangles are ſix.

CASE I. Given two ſides, and an angle oppoſite to one of them.

CASE II. Given two angles, and a ſide oppoſite to one of them.

CASE III. Given two ſides, and the included angle.

CASE IV. Given two angles, and the included ſide.

CASE V. Given the three ſides.

CASE VI. Given the three angles.

Theſe ſix caſes include all the variety that can ariſe in ſpherical triangles.

IN the following ſolutions, are given three conſtructions to every caſe, whereby each ſide is laid on the plane of projection, or (as it is commonly called, the) primitive circle.

To abbreviate the directions given in the following conſtructions, it is to be underſtood, that the primitive circle is always firſt deſcribed, and two diameters drawn at right angles.

THE ſector is alſo ſuppoſed to be ſet to the radius wanted, on the ſcale uſed; and the tranverſe diſtance of the degrees propoſed is to be taken from the chords, or ſecants, or tangents, &c. according to the name mentioned in the conſtruction.

So-

## SOLUTION *of* CASE I.

EXAM. In the ſpherical triangle ABD.
Given AB = 29° 50′
DB = 63 59
∠D = 25 55
Required the triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 1. Pl. VII.

1ſt. Make DB = chord of 63° 59′, and draw the diameter BE.

2d. From D, with the ſecant of the ∠ D, 25° 55′, cut the diameter ⊙ 1 in C: on C as a center, with that radius, deſcribe the circumference DA, and the angle BDA will be 25° 55′.

3d. Make B*d* equal to AB, with the chord of 29° 50′.

4th. With the tangent of AB, 29° 30′, from *d*, cut ⊙ B produced in *b*; and from *b*, with that radius, cut DA in A or *a*.

5th, Through B, A, E, deſcribe a circumference, and the triangle BDA will be that required; whoſe parts DA, ∠B, and ∠A may be thus meaſured.

*To meaſure* DA.

6th. Make ⊙ P equal to the tangent of half the angle BDA, *viz.* 12° 57½′; then a ruler on P and A, gives *e*; and D *e* meaſured on the chords, gives the degrees in DA, *viz.* 42° 9′.

*To meaſure* ∠B.

7th. Draw the diameter FG at right angles to BE, cutting the circumference BAE in *s*; A ruler by B & *s* gives *f*; make *fg* equal to the chord of 90 deg.

a ruler on *g* and B, gives *p* in the diameter FG. Then E *g* on the chords gives the angle B = 36° 9′.

### *To meaſure* ∠A.

8th. A ruler on A and P, gives *n*; and on A and *p*, gives *m*; and *nm* meaſured on the chords, gives 52° 9′, for the ſupplement of the angle DAB, which is 127° 51′.

### II. *To put* DA *on the primitive circle*, Fig. 2. 1.

1ſt. With the ſecant of the angle D, 25°, 55′, from D, cut the diameter in C; and on C, with the ſame radius, deſcribe the arc DB, and the angle BDA will be 25°, 55′.

2d. Make ⊙ P, equal to the tangent of half the angle D; *viz.* 12° 57′ ½.

3d. On the primitive circle, make D *d* equal to the given ſide DB, with the chord of 63° 59′.

4th. A ruler on B and *d*, gives B; then will BD = 63° 59′.

5th. Draw ⊙ B *r*, cutting the primitive circle in *r*.

6th. Make *r x* = the chord of 90°; or twice the chord of 45°.

7th. A ruler on *x* and B, gives *m* on the primitive circle.

8th. Make *m q* = *mp* = chord of 29° 50′.

9th. A right line through *x* & *p*, *x* & *q*, gives *f* & *e* in ⊙ *r*.

10th. On *fe* as a diameter, deſcribe a circumference, cutting the primitive circle in A, *a*.

11th. A ruler on A & ⊙, gives F.

12th. Through A, B, F, deſcribe a circumference, and the triangle ABD is conſtructed with DA on the primitive circle as required.

### III. *To put* AB *on the primitive circle.* Fig. 3. 1.

1ſt. Make AB = the chord of 29° 50′; and draw the diameter BF.

2d. In A *b* drawn perpendicular to AG, take A *b* = ſine of AB 29° 50′.

3d. Make the angle *b* A *g*, = ∠ D 25° 55′; from A draw A *e* at right angles to A *g*, and from *d*, the middle of A *b*, draw *de* perpendicular to A *b*, cutting A*e*, in *e*; from *e*, with the radius *e* A, deſcribe a circumference A *f b*.

4th. From *b*, with the ſine of BD, 63° 59′, cut the circumference A*fb* in *f*; and draw A *f*.

5th. From A, draw AC at right angles to *f* A, meeting E ⊙ (perpendicular to A ⊙,) continued, in C; and on C, with the radius CA, deſcribe a circumference ADG.

6th. Make B*m* = BD, with the chord of 63° 59′; from *m*, with the tangent of 63° 59′ cut ⊙ B produced, in *n*; on *n*, with the ſame radius, cut ADG in D.

7th. Through B, D, F, deſcribe a circumference, and the triangle ABD will be that which was required.

*Computation by the logarithmic ſcales.*

*To find the angle* A.

The ſines of the angles of ſpheric triangles are as the ſines of their oppoſite ſides.

Then the extent of the compaſſes on the line of ſines from 29° 50′ (= AB) to 25° 55′ (= ∠ C); will reach from the ſine of 63° 59′ (= CB) to the ſine of 52° 9′ (= ∠ A).

But by conſtruction the ∠ A is obtuſe; therefore 127° 51′ (the ſupplement of 52° 9′) is to be taken for the angle A.

*To find the angle* B.

Say, as radius, to the coſine of CB.
So tang. ∠ C, to the cotang of a fourth arc.
And as tang. AB, to the tang. of CB.
So coſine of the 4th arc, to the coſine of a 5th arc.

Then the difference between the 4th and 5th arcs gives ∠ B.

The extent from the ſine of 90° to the ſine of 26° 1′ (= comp. of 63° 59′), will, on the tangents, reach from 25° 55′ to 12° 2′ : But the 4th arc is to be a cotangent; therefore 77° 58′ (the comp. 12° 2′) is the 4th arc.

The extent from the tangent of 29° 50′ to the tangent of 63° 59′, will reach on the line of ſines from 12° 2′ (= comp, of 77° 58′) to 48° 9′.

But the 5th arc is to be a coſine; therefore 41° 51′ (the comp. of 48° 9′) is the fifth arc.

And 41° 51′ taken from 77° 58′ leaves 36° 7′ for the angle B.

The extent from the tangent of 29° 50′ to the tangent 63° 59′ is thus taken. Set one foot on the tangent 29° 50′, and extend the other to the tangent of 45°: Apply this extent on the tangents from 63° 59′ towards the left; reſt the left hand foot, and extend the other to 45°, and the compaſſes will then have the required extent.

### *To find* AC.

Say, as radius, to the coſine of the angle C.

So is the tangent of CB, to the tangent of a 4th arc.

And as coſine of CB, to the coſine of AB.

So is the coſine of the 4th arc, to the coſine of the 5th arc.

Then the difference between the 4th and 5th arcs will give the ſide AC.

The extent on the ſines from 90° to 64° 5′ (the comp. of 25° 55′) will reach on the tangents from 63° 59′ towards the right to 61° 31′ the 4th arc.

Alſo the extent on the ſines from 26° 1′ (= comp. of 63° 59′) to 60° 10′ (= comp. of 29° 50′) will reach from the ſine of 28° 29′ (the complement of 61° 31′) to the ſine of 70° 37′.

But the 5th arc is to be a coſine, therefore 19° 23′ is the 5th arc.

And 19° 23′ taken from 61° 31′ leaves 42° 8′ for the ſide AC.

## SOLUTION *of* CASE II.

EXAM. In the ſpherical triangle ABD.

Given AD = 42° 9′
∠A = 127 50′
∠B = 36 8
Required the triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 2. Pl. VII.

1ſt. From B, with the ſecant of ∠ B, 36° 8′, cut the diameter ⊙ E in C; on C, with the ſame radius, deſcribe the circumference B*a*F: then the angle DBF = the given ∠ B.

2d. Make the angle *n a q* equal to 37° 50′, the difference between 127° 50′ and 90°.

3d. Make *a q* = tangent of DA, 42° 9′; on ⊙ with the ſecant of 42° 9′ deſcribe an arc *q*Q: on C with C *q*, cut the arc *q* Q in Q.

4th. Draw Q ⊙ G cutting the primitive circle in D, and BD will be a ſide of the triangle.

5th. From Q with Q*a*, cut B*a*F in A; and through D, A, G, deſcribe a circumference, and the triangle BAD is that required. Whoſe parts BD, BA and ∠ D are thus meaſured.

6th. BD meaſured on the chords, gives 64 degrees.

7th. Make ⊙ P = tangent of half ∠ B, *viz.* 18° 4′; a ruler on P and A gives *x*; then B*x* meaſured on the chords gives 29° 50′, for BA.

8th. Draw a diameter perpendicular to GD, cutting the circumference DAG in *s*; a ruler on D and *s* gives *m*; make *mn* 90 degrees, then G *n* meaſured on the chords, gives 25° 55′ for the ∠ D.

II. *To put* AB *on the primitive circle.* Fig. 2. 2.

1ſt. From A, with the ſecant of the ſupplement of the ∠A, *viz.* 52° 10′, cut the diameter ⊙F continued in C; on C, with the ſame radius, deſcribe a circumference A*a*E.

2d. Make ⊙ P = the tangent of half the ſupplement of ∠ A, *viz.* 26° 5′; and make A *x* = chord of AD, 42° 9′: a ruler on P and *x*, gives D; then is AD equal to 42° 9′.

3d. On ⊙, with the tangent of the angle B, 36° 8′, deſcribe an arc *mc*; on D, with the ſecant of ∠ B, 36° 8′, cut the arc *mc* in *c*; on *c*, with the ſame radius, deſcribe a circumference DB, then the triangle ADB, will be that required.

III. *To put* DA *on the primitive circle.* Fig. 3. 2.

1ſt. Lay down AD with the chord of 42° 9′: Draw the diameter DF; and another ⊙H, perpendicular to DF.

2d. On A, with the ſecant of the ſupplement of ∠ A, *viz.* 52° 10′, cut the diameter E ⊙ in C; and on C, with the ſame radius, deſcribe the circumference ABG.

3d. Make ⊙ P equal to the tangent of half the ſupplement ∠ A, *viz.* 26° 5′, a ruler by G and P gives *x*.

4th. Make *xm* = *xn* with the chord of ∠B, 36° 8′; a ruler by G and *n* gives *r*, by G and *m* gives *s*; on *b* the middle of *rs*, with the radius *bs*, cut ⊙ H in *p*.

5th. A ruler on F and *p*, gives *h*; make *hk* = *h*D; a ruler or F and *k* gives *c*; with the radius *c* D, deſcribe the circumference DBF; and the triangle ABD, is that ſought.

### *Computation by the Logarithmic Scales.*

### *To find the ſide* BD.

Say, as the ſine of ∠ B, is to the ſide AD.

So is the ſine of ∠ A, to the ſide BD.

Then the extent from the ſine of 36° 8′ to the ſine of 42° 9′, will reach from the ſine of 52° 10′ (the ſupplement of 127° 50′) to the ſine of 63° 59′ = ſide BD.

### *To find the ſide* AB.

Say, as radius, is to the coſine of the ∠ A.

So is the tangent of AD, to the tangent of a 4th arc.

And, as tangent of ∠B, to the tangent of the ∠A.

So is the ſine of the 4th arc, to the ſine of a 5th arc.

Then the difference between the 4th and 5th arcs will be equal to the ſide AB.

The extent from the radius, or the ſine of 90° to the ſine of 37° 50′ (the complement of 52° 10), will reach on the tangents from 42° 9′ to 29° 02′ = 4th arc.

And the extent from the tangent of 36° 8′ to the tangent of 52° 10′, will reach on the ſines from 29° 02′ to 58° 54′ = 5th arc.

Then the difference between 58° 54′ and 29° 02′ gives 29° 52′ for the ſide AB.

The extent from the tangent of 36° 8′ to the tangent of 52° 10′ is taken as ſhewed in the ſecond operation of the firſt caſe.

### *To find the* ∠D.

Say, as radius, is to the coſine of AD.

So is the tangent of ∠ A, to the tangent of a 4th arc.

And as the coſine of ∠ A, to the coſine of ∠ B;

So is the ſine of the 4th arc, to the ſine of the 5th arc.

Then the difference between the 4th and 5th arcs will give the ∠D.

Now the extent from the ſine of 90° to the ſine of 47° 51′ (the complement of 42° 09′). will reach from the tangent of 52° 10′ to the tangent of 43° 40′. But the 4th arc being a cotangent will be 46° 20, the complement of 43° 40′.

Alſo the extent from the ſine of 37° 50′ (the complement of 52° 10′) to the ſine of 53° 52′ (the compliment of 36° 08′), will reach from the ſine of 46° 20′ to the ſine of 72° 15′ the 5th arc.

Then the difference between 72° 15′ and 46° 20′ *viz.* 25° 55′ will be the angle D.

In applying the firſt extent, *viz.* from the ſine of 90° to ſine of 47° 51′, to the tangents; ſet one foot on the tangent of 45° and let the other foot reſt where it falls; move the foot from 45° to 52° 10′; then this extent will reach from 45° to 43° 40′.

## SOLUTION *of* CASE III.

Ex. In the ſpherical triangle ABD.
Given AB = 29° 50′.
BD = 63 59.
∠B = 36 8
Required the triangle.

I. *To put* AB *on the primitive circle.* Fig 1. 3. Pl. VII.

1ſt. Make AB = chord of 29° 50′, draw the diameter EF, and another ⊙ E perpendicular thereto.

2d. From B, with the ſecant of ∠B, 36° 8′ cut ⊙ E in C, the center of BDF.

3d. From ⊙, with the tangent of half ∠ B, *viz.* 18° 4′, cut ⊙ E in P, the pole of BDF.

4th. Make B $x$ = BD, 63° 59′; a ruler on P and $x$, gives D. Through A, D, G, deſcribe a circumference, and the triangle ADB is that required, whoſe parts AD, ∠ A, and ∠ D may be thus meaſured.

5th. A ruler on A and $s$ gives $z$, make $zy$ = chord of 90°; a ruler on A and $y$ gives $p$ the pole of A $s$ G; a ruler on $p$ and D, gives $n$, and A $n$ meaſured on the chords gives 42° 8′ for AD.

6th. G $y$ meaſured on the chords, gives 52° 11′ for the ſupplement of ∠ A; therefore ∠ A = 127° 49′.

7th. A ruler on D and $p$ gives $r$, on D and P, gives $m$; and $rm$, meaſured on the chords gives 25° 56′ for the angle BDA.

### II. *To put* DB *on the primitive circle.* Fig. 2. 3.

1ſt. Make DB = chord of 63° 59′: draw the diameter BF and perpendicular thereto, the diameter ⊙ G.

2d. From B, with the ſecant of ∠ B, 36° 8′, cut ⊙ G in C; on C with CB, deſcribe the circumference BAF.

3d Make ⊙ P = tangent of half ∠ B, 18° 4′, and D $x$ = chord of AB 29° 50′, a ruler on P and $x$ gives A; through D, A, E, deſcribe a circumference, and the triangle ABD is that required.

### III. *To put* AD *on the primitive circle.* Fig. 3. 3.

1. In a right line $ed$, touching the primitive circle in any point $b$, take $bd$ = tangent of BD, 63° 59′; and $be$ = tangent of AB, 29° 50′.

2. Make the angle $dba$ = ∠ B, 36° 8′, and make $ba = be$.

3. From $d$, ⊙, with D$a$, ⊙$e$, deſcribe arcs croſſing in $x$; from $x$, $d$, draw the diameters AE, DF; and others OG, OH, perpendicular to AE, FD.

4. From $d$, $x$, with $bd$, $eb$, deſcribe arcs croſſing in B; and draw $d$B, $x$B.

5. From B draw BC, perpendicular to $x$B, and meeting ⊙G produced in C; alſo draw B$c$ perpendicular to $d$B, and meeting ⊙H in $c$; then C is the center of a circumference through A, B, E; and $c$ the center of that through D, B, F; and the triangle ABD is that required.

### *Computation by the Logarithmic Scales.*

### *To find the angles* A *and* C.

Say, as the ſine of half the ſum of the given ſides
To the ſign of half their difference;
So is the cotangent of half the given angle
To the tangent of half the difference of the required angles.

And, as the coſine of half the ſum of the given ſides
To the coſine of half their difference;
So is the cotangent of half the given angle
To the tangent of half the ſum of the required angles.

Then the half difference of the required angles added to their half ſum will give the greater angle A.

And the half difference of thoſe angles taken from their half ſum will give the leſſer angle D.

Now the ſum of the given ſides 63° 59′ and 29° 50′ is 93° 49′, their difference is 34° 09′; the half ſum = 46° 54½′, and the half difference is 17° 04½′.

Alſo half the given angle B is 18° 04′.

Then the extent from the ſine of 46° 54′ to the ſine of 17° 4′, will reach from the tangent of 71° 56′ (the complement of 18° 4′) to the tangent of 50° 57′ the half difference of the required angles.

Here the extent on the ſines is from right to left or decreaſing; ſo the extent on the tangents muſt be from left to right, which in this caſe is decreaſing.

Alſo the extent from the ſine of 43° 6′ (the complement of 46° 54′) to the ſine of 72° 56′ (the complement of 17° 04′), will on the ſcale of tangents reach from 71° 56′ (the complement of 18° 4′) to 76° 53′ the half ſum of the required angles.

Then the ſum of 76° 53′ and 50° 57′ = 127° 50′ = ∠A.

And the difference of 76° 53′ and 50° 57′ = 25° 56′ = ∠C.

The angles being known, the other ſide may be found by oppoſite ſides and angles, and is 42° 08′.

Or the other ſide may be found without knowing the angles.

Say, as radius is to the coſine of the given angle;

So is the tangent of either given ſide, to the tangent of a 4th arc.

Which 4th arc will be like the ſide uſed when the given angle is acute, otherwiſe it will be of a contrary kind with the ſide uſed.

Then take the difference between the 4th arc and the other given ſide, call the remainder a 5th arc.

And as the coſine of the 4th arc is to the coſine of a 5th arc;

So is the coſine of the ſide uſed in the former proportion

To the coſine of the ſide required.

Now the extent from the ſign of 90° to the ſine of 53° 52′ (= complement of 36° 08′) will reach from the tangent of 29° 50′ to the tangent of 24° 51′ the 4th arc.

And 24° 51′ taken from 63° 59′ leaves 39° 8′ for the 5th arc.

Then the extent from the ſine of 65° 09′ (the complement of 24° 51′, to the ſine of 50° 52′ (the complement of 39° 08′) will reach from the ſine of 60° 10′ (the complement of 29° 50′) to the ſine of 47° 51′; whoſe complement, *viz.* 42° 09′ is the ſide required.

## SOLUTION *of* CASE IV.

Ex. In the ſpherical triangle ABD:

Given ∠ D = 25° 55′.
∠ B = 36° 08′.
DB = 63° 59′,
Required, The triangle.

I. *To put* DB *on the primitive circle.* Fig. 1. 4. Pl. VII.

1. Make DB = chord of 63° 59′; draw the diameter BF, and draw CG perpendicular to BF.

2 From B, with the ſecant of ∠ B, 36° 8′, cut ⊙G in C; and C will be the center of BAF.

3. From D, with the ſecant of ∠ D, 25° 55′; cut ⊙H in $c$, and $c$ will be the center of DAE; and the triangle DAB is that which was required; whoſe parts DA, BA, and ∠A, are thus meaſured.

4. Make ⊙$p$ = tangent of $\frac{1}{2}$ ∠D, 12° 57$\frac{1}{2}$′, a ruler on $p$ and A gives $x$; then D$x$ meaſured on the chords gives 42° 10′ for AD.

5. Make ⊙P = tangent of $\frac{1}{2}$ ∠B 18° 4′, a ruler on P and A, gives $z$; then B$z$ meaſured on the chords, gives 29° 54′ for AB.

6. A ruler on A and $p$, gives $n$, on A and P, gives $m$; and $nm$ meaſured on the chords gives 52° 10′ the ſupplement of the angle A. Therefore ∠A = 127° 50′.

II. *To put* DA *on the primitive circle.* Fig. 2. 4.

1ſt. From D, with the ſecant of ∠D, 25° 55′; cut ⊙F in C; and C is the center of the circumference DBE.

2d. Make ⊙P = tangent of $\frac{1}{2}$ ∠D, 12° 57$\frac{1}{2}$; and make D$x$ = chord of BD, 63° 59′; a ruler on P, $x$, gives B; and DB is 63° 59′.

3d. Make the angle CB$c$ = ∠ B, 36° 8′; through C, draw $mc$ perpendicular to B⊙, cutting B$c$ in $c$; on $c$, with the radius $c$B, deſcribe the circumference ABG; and the triangle ABD, is that which was required.

III. *To put* AB *on the primitive circle.* Fig. 3. 4.

1ſt. From B, with the ſecant of ∠B, 36° 8′ cut ⊙F in C; and C is the center of the circumference of BDE.

2d. Make B$x$ = chord of BD, 63° 59′; and ⊙P = tangent of $\frac{1}{2}$ ∠ B, 18° 4′; a ruler on P and $x$ gives D; then is BD = 63° 59′.

3d. Make the angle CD$c$ = ∠D, 25° 55′; then $mc$ drawn perpendicular to ⊙ D, meeting D$c$ in $c$, gives $c$ the center of the circumference ADG; and the triangle ABD will be that required.

### *Computation by the Logarithmic Scales.*

### *To find the angle* A.

Say, as radius is to the coſine of the given ſide;

So is the tangent of either given angle to the cotangent of a 4th arc,

Call the difference between the other given angle and the 4th arc, the 5th arc.

And, as the ſine of the 4th arc, is to the ſign of the 5th arc;

So is the coſine of the angle uſed in the former proportion

To the coſine of the required angle.

The 4th arc will be of the ſame kind with the angle firſt uſed if the given ſide is leſs than 90°; but of a contrary kind if that ſide is greater than 90°.

Arcs are ſaid to be of the ſame kind, when both are leſs, or both greater, than 90 degrees.

The required angle will be of the ſame kind with the angle uſed in the proportions, if the 4th arc is leſs than the other angle; but of an unlike kind when the 4th arc is greater than the other angle.

Now the extent from the ſine of 90° to the ſine of 26° 01′ (the complement of 63° 59′) will reach from the tangent of 25° 55′ to the tangent of 12° 02′: But this is the complement of the 4th arc, which is 77° 58′.

And 36° 08′ taken from 77° 58′ leaves 41° 50′ for the 5th arc.

Then the extent from the ſine of 77° 58′ to the ſine of 41° 50′, will reach from the ſine of 64° 5′ (the complement of 25° 55′) to the ſine of 37° 50, which is the complement to 52° 10′.

But as the 4th arc was greater than 36° 08′, the angle ſought is to be of a contrary kind to 25° 55′ (= ∠D), that is, that A is to be obtuſe; ſo 127° 50′ (the ſupplement of 52° 10′) is to be taken for the angle A, Now all the angles and one ſide being known, the other ſides may be found by the proportion ſubſiſting between the ſines of angles, and the ſines of their oppoſite ſides.

Or ſay,

As the ſine of half the ſum of the given angles
Is to the ſine of half the difference of thoſe angles;
So is the tangent of half the given ſide
To the tangent of half the difference of the required ſides.

And

As the coſine of half the ſum of the given angles
Is to the coſine of half the difference of thoſe angles;
So is the tangent of half the given ſide
To the tangent of half the ſum of the required ſides.

Then the half difference added to the half ſum gives the greater of the ſought ſides.

And the half difference ſubtracted from the half ſum gives the leſſer of the ſought ſides.

Now the half ſum of the given angles, *viz.* $\frac{1}{2}\angle D + \frac{1}{2}\angle B = 31^\circ\ 01\frac{1}{2}'$,

And the half difference of thoſe angles, *viz.* $\frac{1}{2}\angle B - \frac{1}{2}\angle D = 5^\circ\ 6\frac{1}{2}'$,

Alſo the half of the given ſide DB, is $31^\circ\ 59\frac{1}{2}'$.

Then the extent from the ſine of 31° 1′, to the ſine of 5° 6′;

Will reach from the tangent of 31° 59′, to the tangent of 6° 3′.

And the extent from the ſine of 58° 59 (= complement of 31° 01′) to the ſine of 84° 54′ (the complement of 5° 6′), will reach from the tangent of 31° 59′ to the tangent of 35° 58′.

Then the ſum of 35° 58′ and 6° 3′, *viz.* 42° 01′ = AD.

And the difference of 35° 58′ and 6° 3′, *viz.* 29° 55′ = AB.

## SOLUTION *of* CASE V.

Ex. In the ſpherical triangle ABD.
Given AB = 29° 50°
AD = 42 9
BD = 63 59
Required, The triangle.

I. *To put* AB *on the primitive circle.* Fig. 1. 5. Pl. VII.

1ſt. Make AB = chord of 29° 50′; draw the diameter BF.

2. Make A*n* = chord of AD, 42° 9′; and B*m* = chord of BD, 63° 59′.

3d. From *n*, with the tangent of AD, 42° 9′, cut EA produced in C; and from C, with that radius, deſcribe the arc *nn*; from *m*, with the tangent of BD, 63° 59′, cut FB produced in *c*; and from *c*, with the radius *cm*, cut the arc *nn* in D.

4th. Through A, D, E; B, D, F, deſcribe circumferences, and the triangle ADB is that which was required; whoſe angles A, B, D, are thus meaſured.

5th. A ruler on A and *a*, gives *x*; on B and *b*, gives *z*; make *xy*, *zv*, each 90°; a ruler on A and *y* gives P, in a radius perpendicular to AE; and a ruler on B and *v* gives *p*, in a radius perpendicular to BF.

6th. E*y* meaſured on the chords, gives 52° 12′ for the ſupplement of the ∠A; therefore ∠A = 127° 48′.

7th. F*v* meaſured on the chords, gives 36° 10′ for the angle B.

8th. A ruler on D and P gives *t*, and on D and *p* gives *s*; then *ts* meaſured on the chords, gives 25° 58′ for the angle D.

The ſides AD, DB, are put on the primitive circle, by a conſtruction ſo like the foregoing one, that it is needleſs to repeat it. See figures 2. 5. and 3. 5.

### *Computation by the Logarithmic Scales.*

### *To find the angle* A.

The ſides including the angle A are AD = 42° 09′
And AB = 29 50
Their difference call X = 12 19
The ſide oppoſite the ∠A is BD = 63 59

Then the ſum of BD and X is 76° 18′; the half ſum is 38° 09′;

And the difference of BD and X is 51° 40′; the half difference is 25° 50′.

Now take the extent on the line of ſines, from the half ſum 38° 9′ to either of the containing ſides, as to 29° 50′; apply this extent from the other containing ſide 42° 09′ towards the left, there let the foot reſt, and extend the other point (*viz.* that which was ſet on 42° 09′) to the half difference 25° 50′; then this extent applied to the line of verſed ſines, will reach from 0 degrees (at the beginning) to 52° 12′; the ſupplement of which, or 127° 48′ will be the degrees in the angle A.

### Again. *To find the angle* D.

The ſides including the angle A, are BD = 63° 59′
And AD = 42 09
Their difference call X = 21 50
The ſide oppoſite to the ∠D is AB = 29 50

Then the ſum of AB and X is 51° 40′; the half ſum is 25° 50′.

And the difference of BA and X is 8° 0′; the half difference is 4° 00′.

Then the extent on the ſines from 25° 50′ to 63° 59′ will reach from the ſine of 42° 09′ to ſome point beyond 90°; therefore apply the extent between 25° 50′ and 63° 59′ from the ſine of 90° downwards, let the point reſt where it falls, and bring that point which was ſet on 90° to 42° 09′; then will the diſtance between the feet ſhew how far the firſt extent would reach beyond 90°: Now apply this extent on the ſines from the point oppoſite to the middle I on the line of numbers, the other foot falling upwards to the right, let it reſt there, and extend the other foot to the half difference 4° 0′: Then this extent applied to the verſed ſines, one foot being ſet on the point oppoſite the middle I on the line of numbers, the other foot will fall on 154° 5′; the ſupplement whereof, *viz.* 25° 55′ will be the angle D.

### SOLUTION *of* CASE VI.

Ex. In the ſpherical triangle ABD:

Given ∠A = 127° 50′.
∠B = 36° 8′.
∠D = 25° 55′.
Required, The triangle.

I. *To put* AB *on the primitive circle.* Fig. 1. 6. Pl. VII.

1ſt. From B, with the ſecant of ∠B, 36° 8′, cut ⊙F in C, and C will be the center of the circumference through B, D, E.

2d. From ⊙, with the tangent of 52° 10′ the ſupplement of ∠A, deſcribe an arc *xc*.

3d. Make an angle C*aq* = ∠D, 25° 55′; make *aq* equal B*x*. (= ſecant of 52° 10′.)

4th. From C, with the radius C*q*, cut *xc* in *c*; From *c*, with the radius *qa*, deſcribe a circumference ADG; and the triangle ABD, is that which was required: whoſe ſides AB, BD, DA, are meaſured as follows.

5th. A ruler on B and *a* gives *d*, and on A and *b*, gives *f*; make *dg*, *fh*, each 90 degrees; a ruler on *g* and B gives P, and on *h* and A, gives *p*, in ⊙F, ⊙H, drawn perpendicular to BE, AG.

6th. A ruler on P and D gives *n*, and on *p* and D, gives *m*.

7th. Then BA, B*n*, A*m*, meaſured on the chords, gives 29° 50′; 63° 59′; 42° 9′; for the reſpective meaſures of BA, BD, AD.

The directions for this conſtruction, may be eaſily applied to the putting either of the other ſides on the primitive circle. Fig. 2. 6. and 3. 6. Pl. VII.

### *Computation by the Logarithmic Scales.*

#### *To find the ſide* BD:

The angles including the ſide BD, are ∠B = 36° 08′
And ∠D = 25 55
Their difference call x = 10 13
The ſupplement of the ∠ oppoſite to BD is 52 10

The ſum of the ſupplement of ∠A and x is 62° 23′; the half ſum is 31° 11½′.

The difference of the ſupplement of ∠A and x is 41° 57′; the half difference is 20° 58½′.

Now on the ſines, the extent from the half ſum 31° 11½′ to 25° 55′ will reach from 36° 08′ to a fourth ſine; and the extent from that fourth ſine to the ſine of the half difference 20° 58½′ will reach on the verſed ſines from the beginning to about 64° the ſide ſought.

And in like manner may the other ſides be found.

## SECT. XIX.

### *Of the proportional Compaſſes.*

THOSE compaſſes are called proportional, whoſe joint lies between the points terminating each leg; in ſuch a manner, that when the compaſſes are opened, the legs form a croſs.

SUCH compaſſes are either ſimple or compound.

Simple proportional compaſſes, are ſuch, whoſe center is fixed: One pair of theſe, ſerve only for one proportion.

Thus, if a right line is to be divided into 2, 3, 4, 5, *&c.* equal parts; or the cord of $\frac{1}{3}$, $\frac{1}{5}$ $\frac{1}{7}$, *&c.* part of a circumference is to be taken; there muſt be as many of ſuch compaſſes, as there are diſtinct operations to be performed.

In each caſe, take the length of the right line, or of the radius of the circle, between the longer points of the legs; and the diſtance of the ſhorter points will be the part required.

Compound proportional compaſſes, are thoſe wherein the center is moveable; ſo that one pair of theſe will perform the office of ſeveral pairs of the ſimple ſort.

In the ſhanks of theſe compaſſes are grooves, wherein ſlides the center, which is made faſt by a nut and ſcrew.

On each ſide of theſe grooves, ſcales are placed; which may be of various ſorts, according to the fancy of the buyer: But the ſcales which the inſtrument-makers commonly put on theſe compaſſes, are only two, *viz.* lines and circles.

By the ſcale of lines, a right line may be divided into a number of equal parts, not exceeding the greateſt number on the ſcale; which is generally 12.

Exam. I. To divide a given right line, (ſuppoſe of 7½ inches long,) into a propoſed number of equal parts. (as 11.)

Operation. Shut the compaſſes; unſcrew the button; move the ſlider until the line acroſs it, coincides with the 11th diviſion on the ſcale of lines; ſcrew the button faſt; open the compaſſes, until the given line can be received between the longer points of the legs; then will the diſtance of the ſhorter points be the 11th part of the given line, as required.

By the ſcale of circles, a regular polygon may be inſcribed in a given circle; provided the number of ſides in the polygon, do not exceed the numbers on the ſcale, which commonly proceed to 24.

Exam. II To inſcribe in a circle of a known radius, (ſuppoſe 6 Inches) a regular polygon of 12 ſides?

Operation. Shut the compaſſes; unſcrew the button; ſlide the center until its mark coincides with the 12th diviſion on the ſcale of circles; ſcrew the button faſt; take the given radius between the longer points of the legs; then will the diſtance of the ſhorter points, be the ſide of the polygon required.

These ſcales are applicable to ſeveral other uſes beſide the foregoing ones, in the ſame manner, as the like lines on the ſector are.

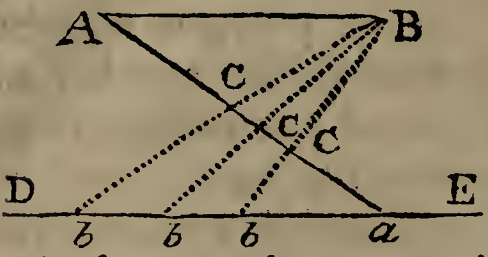
From theſe operations it is evident, that the lengths of the longer and ſhorter legs, (reckoned from the center,) muſt always be proportional to the diſtance of their extremities.

Therefore, to divide a right line into 2, 3, 4, 5, 6, 7, 8, *&c.* equal parts; the lengths of each leg, from the center, will be expreſſed by the following ſeries, the whole length of the inſtrument being taken for unity.

Longer leg $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, *&c.*
Shorter leg $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, *&c.*

These diviſions may be very accurately laid on the legs of the compaſſes by the help of a good ſector. (See Prob. 14.)

Or, the diviſions of this ſcale of lines may be found by the following conſtruction.



DRAW the indefinite right line DE; and from any point A, without DE, draw A*a*, equal to the ſhank of the compaſſes, making any angle at *a*, with DE. Through A draw the right line AB, parallel to DE, and equal to the given line from whoſe parts the proportions are taken.

LET A*a* contain N parts.

Now that *ab* may be the *nth* part of AB, or, that AB may be *n* times *ab*.

LET $ac = \frac{1}{n+1}$ N, or $AC = \frac{1}{n+1}$ N; then the point C is the center of the ſcrew pin. And through C, drawing BC, meeting DE in *b*; then is $ab = \frac{1}{n}$ of AB, or AB $= n$ times *ab*.

For $\frac{ab}{AB} = \frac{ac}{AC} = \frac{n}{1}$.

If the center of the ſcrew-pin be diſtant from the mark in the ſlider, the $\frac{1}{m}$ part of N.

Then $ac = \frac{m \pm s}{s} \times \frac{N}{m}$ (putting $s = n + 1$.)

Ex. If $N = 10000$, $m = 400$, and $n = 1$, or 2, or 3, &c.

Then $ac = 5000$, or 3333, or 2500, &c. when the diviſions on the ſhank reſpect the center pin.

And $ac = \left\{ \begin{matrix} 5025 \text{ or } 3358 \text{ or } 2525, \&c. \\ 4975 \text{ or } 3308 \text{ or } 2475, \&c. \end{matrix} \right\}$ when the diviſions reſpect a mark in the ſlider, diſtant from the center pin, $\frac{1}{25}$ of the length of the Inſtrument.

THE ſcale for dividing of the circle, or the diviſions for regular polygons may be found thus.

FIND the angles at the center, of as many regular polygons as are to be deſcribed on the compaſſes.

SEEK the ſines belonging to the half of each angle, to the radius 1.

To each of theſe ſines doubled, add the radius 1.

Then will the reciprocal of theſe numbers, be the lengths of the polygonal diviſions, on the legs of the compaſſes, reckoned from the longer point; the length of the inſtrument being accounted unity.

For the longer and ſhorter legs, (or points) are in the ſame ratio, as are the radius and chord of the angle at the center.

And as the ſum of the radius and chord, is to the radius; ſo is the ſum of the longer and ſhorter legs, (or points) to the length of the longer point.

And hence was the following table compoſed, which ſhews the decimal parts on the leg, from the longer point to the center.

| N° Sides. | Length on the Leg. | N° Sides. | Length on the Leg. | N° Sides. | Length on the Leg. |
|---|---|---|---|---|---|
| 3 | 0,3333 | 11 | 0,6396 | 19 | 0,7523 |
| 4 | 0,4142 | 12 | 0,6589 | 20 | 0,7617 |
| 5 | 0,4597 | 13 | 0,6763 | 21 | 0,7706 |
| 6 | 0,5000 | 14 | 0,6921 | 22 | 0,7785 |
| 7 | 0,5354 | 15 | 0,7063 | 23 | 0,7860 |
| 8 | 0,5665 | 16 | 0,7193 | 24 | 0,7931. |
| 9 | 0,5940 | 17 | 0,7313 | | |
| 10 | 0,6180 | 18 | 0,7423 | | |

These diviſions may be truly laid off by the help of a ſector; making the whole length of the proportional compaſſes, a tranſverſe diſtance to 10 and 10, on the line of lines.

The complements, to unity, of the numbers in the table, will give the diſtances of the diviſions from the other point of the inſtrument.

If the mark in the ſlider, is at ſome diſtance from the center, as it commonly is, then this diſtance, which is always known, muſt be added to, or ſubtracted from, the foregoing numbers, according to that ſide of the center the mark is on; and the ſums, or remainders, will give the diſtances of the diviſions from one of the points.

About *Michaelmas*, 1746, was finiſhed a pair of proportional compaſſes, with the addition of a very curious and uſeful contrivance; (ſee the plate fronting the title page) *viz.* into one of the legs (A) at a ſmall diſtance from the end of the groove, was ſcrewed a little pillar *(a)* of about $\frac{1}{3}$ of an inch high, and perpendicular to the ſaid-leg; through this pillar, and parallel to the leg, went a ſcrew pin *(bb)*; to one end of this ſcrew, was ſoldered a ſmall beam *(cc)* nearly of the length of the groove in the compaſſes; the beam was ſlit down the middle lengthwiſe, which received a nut (*f*) that ſlid along the ſlit (*dd*); this nut could be ſcrewed to the beam faſt enough to prevent ſliding; one end (*e*) of the ſcrew of the nut (*f*) falls into a hole in the bottom of the ſcrew to the great nut (*g*) of the compaſſes; the ſcrew pin (*bb*) paſſed through an adjuſter (*h*): To uſe this inſtrument, ſhut the legs cloſe, ſlacken the ſcrews of the nuts *g* and *f*; move the nuts and ſlider *k* to the diviſion wanted, as near as can be readily done by the hand; ſcrew faſt the nut *f*; then by turning the adjuſter *h*, the mark on the ſlider *k*, may be brought exactly to the diviſion; ſcrew faſt the nut *g*; open the compaſſes; gently lift the end *e*, of the ſcrew of the nut *f*, out of the hole in the bottom of the nut *g*; move the beam round its pillar *a*, and ſlip the point *e*, into the hole in the pin *n*; ſlacken the ſcrew of the nut *f*; take the given line between the longer points of the compaſſes, and ſcrew faſt the nut *f*: Then may the ſhorter points of the compaſſes be uſed without any danger of the legs changing their poſition;

this being one of the inconveniencies that attended the proportional compaſſes before this ingenious contrivance; which was made by Mr. *Thomas Heath*, Mathematical Inſtrument-maker in the *Strand*, *London.*

The proportional compaſſes had not been long invented before there were ſeveral learned and ingenious perſons who contrived a great variety of ſcales to be put thereon; but theſe are here omitted, becauſe the credit of the proportional compaſſes is greatly fallen, ſince the invention of the ſector, the latter being a much more uſeful inſtrument than the former, and not ſo ſubject to be put out of order; for if one of the points of theſe compaſſes ſhould be blunted or broke, the inſtrument cannot be uſed, until the damaged point be replaced by a new one. However, thoſe who are deſirous of knowing the conſtruction and uſe of ſuch ſcales on the proportional compaſſes, may be amply ſatisfied in conſulting *Hulſius*, *Horſcher*, *Galgemaire*. *Bion*, and others mentioned in the preface to this book.

# APPENDIX.

CONTAINING

The DESCRIPTION and USE of the

# GUNNERS CALLIPERS.

A PAIR of Callipers is an inſtrument uſed to take the diameters of convex and concave bodies.

THE inſtrument called the Gunners Callipers, conſiſts of two thin rulers or plates, which are moveable quite round a joint, by the plates folding one over the other.

THE length of each ruler or plate is uſually between the limits of ſix inches and a foot, reckoned from the centre of the joint; and from one to two inches broad: But the moſt convenient uſeful ſize is about nine inches long. The figure is beſt ſeen in the plate.

ON theſe rulers are a variety of ſcales, tables, proportions, &c ſuch as are eſteemed uſeful to be known by gunners and other perſons employed about artillery: But except the taking of the calibre of ſhot and cannon, and the meaſuring of the magnitude of *ſaliant* and *entring angles*, there are none of the articles with which the callipers are uſually filled, eſſential to this inſtrument; the ſcales are, or may be, put on the ſector; and the tables, precepts, &c. may be put into a pocket book, where they will not need ſo much contraction: However, for the ſake of thoſe who are deſirous of having a ſingle inſtrument perform many things, the following articles and their diſpoſition on the callipers are here offered: Some of which were propoſed many years ago by my much eſteemed friend Mr. *Charles Labelye*, engineer to the works of *Weſtminſter-bridge.*

*Articles propoſed to be put on the Gunners Callipers.*

I. THE meaſures of convex diameters in inches.

II. THE meaſures of concave diameters in inches.

III. THE weights of iron ſhot from given diameters.

IV. THE weight of iron ſhot proper to given gun bores.

V. THE degrees of a ſemicircle.

VI. THE proportion of Troy and Averdupoiſe weight.

VII. THE proportion of Engliſh and French feet and pounds.

VIII. FACTORS uſeful in circular and ſpherical figures.

IX. TABLES of the ſpecific gravity and weights of bodies.

X. TABLES of the quantity of powder neceſſary for proof and ſervice of braſs and iron guns.

XI. RULES for computing the number of ſhot or ſhells in a finiſhed pile.

XII. RULES concerning the fall of heavy bodies.

XIII. RULES for the raiſing of water.

XIV. THE rules for ſhooting with cannon or mortars.

XV. A LINE of inches.

XVI. LOGARITHMIC ſcales of numbers, ſines, verſed ſines and tangents.

XVII. A SECTORAL line of equal parts, or the line of lines.

XVIII. A SECTORAL line of plans or ſuperficies.

XIX. A SECTORAL line of ſolids.

The Callipers propoſed for the reception of the foregoing articles is nine inches long, and each leg two inches broad at the head, and at the points; part of the breadth between the ends is hollowed away in a curve, in order to contain the curvature of the ball, whoſe diameter is taken between the points; one of ten inches diameter is the largeſt that can conveniently be taken with a nine inch Calliper; thoſe of ſix inches cannot well be applied to a ſhot of more than ſeven inches diameter.

For the eaſe of reference; it will be convenient to diſtinguiſh the four faces of the Callipers by the letters A, B, C, D: Each of the faces A and D conſiſt of a circular head and a leg; the other faces B and C conſiſt only of a leg.

## ARTICLE I.

### *Of the meaſures of convex diameters.*

On part of the circular head joining to the leg of the face A, are diviſions diſtinguiſhed by the title of *ſhot diameters*: Theſe are to ſhew the diſtance in inches, and tenth parts of an inch, of the points of the Callipers when they are opened.

### THE USE.

Open the points of the Callipers ſo, that they may take in the greateſt diameter of the ball; then will the bevil edge marked E ſhew among the foreſaid diviſions, the diameter of that ball in inches and tenth parts, not exceeding ten inches.

*Theſe diviſions may be thus laid down by the ſector.*

Open the ſector until the radius of the circle, whereon is marked the ſcale of diviſions on the head of the Callipers, taken with the compaſſes, falls tranſverſely in the ſcales of lines, on the diviſions ſhewing the diſtance between the centre of the Callipers and its points: Then the tranſverſe diſtances of the ſeveral diviſions on the ſcales of lines, being applied like chords to the circle of diviſions on the head of the Callipers will give the diviſions required.

Thus in the nine inch Callipers, the radius of the head, or circle of diviſions being one inch, and the breadth at the points two inches; the diſtance between the centre and points will be $(\sqrt{82}=)$ 9,055385: Then one inch being made a tranſverſe diſtance on the ſcales of lines to $9\frac{5}{100}$; the tranſverſe diſtances of 10, 9, 8, 7, 6, &c. being applied to the circle on the head of the Callipers appropriated for the ſcale, from the mark where the diviſions commence, will give the ſeveral points, which being cut by the bevil edge E will ſhew how far the points of the Callipers are diſtant.

The workmen generally lay theſe diviſions down by trial.

## ARTICLE II.

### *Of the weights of iron ſhot.*

On the circular bevil part E of the face B, is a ſcale of diviſions denominated by ℔ *weight of ſhot.* Theſe are to ſhew the weights of iron ſhot when the diameter is taken between the points of the Callipers: For then the number cut by the inner edge of the leg A, ſhews the weight of that iron ſhot in pounds averdupoiſe, when the weight is among the following ones, *viz.*

℔. $\frac{1}{2}$. 1. $1\frac{1}{2}$. 2. 3. 4. $5\frac{1}{4}$. 6. 8. 9. 12. 16. 18. 24. 26. 32. 36. 42.

Observing that the figures neareſt the bevil edge anſwer to the ſhort lines; and thoſe figures behind them anſwer to the diviſions marked with the longer ſtrokes or lines.

These diviſions are to be laid down from a table ſhewing the diameters of iron ſhots to given weights. Such a table is computed by knowing the weight of one ſhot of a given diameter: Thus an iron ſhot of four inches diameter is found to weigh nine pounds: Then the weights of ſpheres being to one another as the cubes of their diameters, Say, As 9 ℔ is to 64, the cube of 4.

So is any other weight, to the cube of its diameter.

THEN the cube root being taken will give the diameter.

Now ſetting the points of the Callipers to touch one another, make a mark on the bevil edge E oppoſite to the inner edge of the leg A; and this mark will be the beginning of this ſcale of weights: The other diviſions will be obtained by opening the points of the Callipers to the diſtances reſpecting the weights to be introduced, as ſhewn by the table, and marking the diviſion oppoſite to the inner edge of the leg A.

## ARTICLE III.

### *Of the meaſures of concave diameters.*

ON the lower part of the circular head of the face A, and to the right hand of the diviſions for the diameters of ſhot, is another ſcale of diviſions, againſt which ſtands the words *Bores of Guns.*

### THE USE.

### *To find the calibre, or the diameter of the bore of a cannon.*

SLIP the legs of the Callipers acroſs each other, until the ſteel points touch the concave ſurface of the gun in its greateſt breadth; then will the bevil edge F, of the face B, cut a diviſion in the ſcale ſhewing the diameter of that bore in inches and tenth parts.

IN the nine inch Callipers theſe diviſions may be extended to 18 inches diameter, but 14 inches is ſufficient for both cannon and mortars: And in the ſix inch Callipers a diameter greater than 10 inches cannot be conveniently introduced.

*Theſe diviſions may be thus laid down by the ſector.*

SET one inch the radius of the circle on which the diviſions are to be put, as a tranſverſe diſtance to the diviſion $9\frac{5}{100}$ on the ſcale of lines on the ſector : Set the points of the Callipers together, and make a mark on the circular head where it is then cut by the bevil ege F : Then the ſeveral tranſverſe diſtances taken from the ſector, and applied on the circumference of the circular head of the Callipers, from the ſaid mark, the ſeveral diviſions ſhewing the diſtance of the points of the Calliper are thereby obtained.

WORKMEN find theſe diviſions by actually ſetting the points to the diſtance.

## ARTICLE IV.

### *Of the weights of ſhots to given gun bores.*

WITHIN the ſcales of ſhot and bore diameters on the circular part of the face A are diviſions marked *Pounders.*

### THE USE.

WHEN the bore of a gun is taken between the points of the Callipers, the bevil edge F will cut one of theſe diviſions, or be very near one of them : Then the number ſtanding againſt it will ſhew the weight of iron ſhot proper for that gun ; not exceeding 42 pounds.

THE inner figures $\frac{1}{2}$, $1\frac{1}{2}$, 3, $5\frac{1}{4}$, 8. 12. 18. 26. 36. belong to the longeſt ſtrokes or lines ; and the figures 1. 2. 4. 6. 9. 16. 24. 32. 42 belong to the ſhort ſtrokes.

THE diameters given by theſe pounders are larger than thoſe given for the ſame weights of ſhot ; becauſe there is an allowance made, *called Windage*, that the ſhot may roll eaſily along the chace.

## ARTICLE V.

### *Of the degrees in the ſemicircular head.*

These degrees are placed on the upper half of the circular head of the face A, where are three concentric ſcales of degrees: The outward ſcale has 180 degrees numbered from the right to left, with 10. 20. 30. 40. &c. to 180: The middle ſcale is numbered in the ſame manner, but the contrary way: And the inmoſt ſcale begins in the middle with 0, and is numbered from thence both ways with 10. 20. 30. &c. to 90 degrees.

### THE USE.

### *Firſt to meaſure an entring, or internal, angle.*

Apply the legs of the Callipers ſo that its outſide edges coincide with the legs of the given angle; then will the bevil edge F cut the degrees ſhewing the meaſure of that angle in the outſide ſcale.

### *Secondly. To meaſure a ſaliant, or external, angle.*

Slip the legs of the Callipers acroſs each other, ſo as their outſide edges may coincide with the legs of the given angle; then will the bevil edge E cut the degrees ſhewing the meaſure of that angle: Theſe degrees are to be counted on the middle ſcale.

Hence an angle of any number of degrees may be readily laid down by the Callipers, either on paper, or in the field.

Thus. Open the Callipers, the legs being croſſed, until the edge E cuts the degrees on the middle ſcale; the croſſing edges of the inſtrument will then form the ſides of that required angle: The Callipers then laid flat on the paper or ground, lines drawn by the ſtrait ſides will expreſs that angle.

### *Thirdly. To find the elevation of cannon and mortars, or of any other oblique plane or line.*

Pass one end of a fine thread into the notch on the plate B, and to the other end tie a bullet, or other weight: Apply the ſtrait ſide of the plate A to the ſide of the body whoſe inclination is wanted; hold the plate A in this poſition, and move the plate B until the thread falls upon the line near the centre marked *Perp.* Then will the bevil edge F cut the degrees, counted on the inner ſcale, ſhewing the inclination which that body makes with the horizon.

*Note.* When the edge F cuts 0 on the inner ſcale; and the ſtring cuts the *Perp.* mark, then the ſtrait ſide of the leg A is horizontal: If the head of the Callipers is elevated above the other end, then the edge F muſt ſlide downwards towards the ſtrait ſide of the leg A: But if the head of the Callipers is held lower than the other end, then muſt the edge F ſlide the contrary way.

As the outſide of a cannon or mortar is not parallel to its chaſe; therefore a ſtrait ſtick muſt be applied to the bottom or top of the bore, touching the chaſe; and the ſide of the Callipers be laid on that ſtick.

## ARTICLE VI.

### *Of the proportion of Troy and Averdupoiſe Weights.*

On the face C near the point of the Callipers is a little table ſhewing the number of pounds that are contained in an equal weight expreſſed in pounds Troy; and the contrary.

These numbers are taken from very accurate experiments made in the year 1744 by the late *Martin Folkes*, Eſq; Preſident of the Royal Society, aſſiſted by ſeveral other gentlemen of that learned Body.

## THE TABLE.

| ℔ Troy | ℔ Averd. | oz. Troy | oz. Averd. |
|---|---|---|---|
| 576,00000 | =700 | 82 | =90 |
| 1,00000 | = 0,82274 | 1,00000 | = 1,09707 |
| 1,21545 | = 1,00000 | 0,91152 | = 1,00000 |

## THE USE.

EXAMPLE I. *What weight in pounds Troy is equal to a braſs gun weighing* 18 C *wt.*

Now 18 C wt. is equal to 2016 ℔ (=18×112).
THEN 1 : 1,21545 : : 2016 : 2450 ℔ Troy.
OR, 0,82274 : 1 : : 2016 : 2450 ℔ Troy.
OR, 576 : 700 : : 2016 : 2450 ℔ Troy.

EITHER of theſe methods may be uſed as the operator pleaſes.

EXAMPLE II. *What is the worth of a ton of gold; ſuppoſing* 1 ℔ *Troy makes* 44½ *guineas.*

Now 1 Ton =2240 ℔ Averd. (=20×112).
AND 1 : 1,21545 : : 2240 : 2722,6 ℔ Troy.
ALSO 44½ Guineas, makes 46,725 £ ſterling.
THEN 1 : 2722,6 : : 46,725 : 127213.485 £.
OR, 127213 *l.* 9*s.* 8*d.* ½.

BUT if Troy pounds were given to be converted into Averdupoiſe pounds, then the numbers in the Troy column muſt be the firſt terms of the proportions.

EXAMPLE III. *If a braſs gun weighs* 2450 ℔ *Troy; What is its weight in Averdupoiſe?*

THEN 1 : 0,82274 : : 2450 : 2015,7 ℔ Ave.
OR, 1,21545 : 1,00000 : : 2450 : 2015,7
OR, 700 : 576 : : 2450 : 2016.

ALTHOUGH the Averdupoiſe pound is heavier than the Troy pound, yet the Troy ounce is heavier than the Averdupoiſe ounce, nearly in the proportion of 90 to 82.

Example IV. *In a cheſt of ſilver containing 4380 pieces of eight, each piece weighing $\frac{4}{5}$ of an ounce Troy: How many ounces Averdupoiſe.*

Then 82 : 90 : : $4380 \times \frac{4}{5}$ : 3845,88
Or, 1 : 1,09707 : : $4380 \times \frac{4}{5}$ : 3844,13
Or, 0,91152 : 1 : : $4380 \times \frac{4}{5}$ : 3844,13

Close to the former table is another, ſhewing the number of cubic inches in a gallon, both in wine and beer meaſures; and conſequently their proportions: One uſe is ſhewn by the following Example.

*How long will 33 butts of beer ſerve a crew of 324 men, allowing to each man 3 wine quarts a day?*

Now 33 butts contain 3564 beer gall. ($=108 \times 33$)
And 231 : 282 : : 3564 : $4350\frac{6}{7}$ wine gallons,
And $4350\frac{6}{7}$ gallons makes $17403\frac{3}{7}$ quarts,
Then 17403 divided by 324 gives very near 54.
Consequently $\frac{1}{3}$ of 54, or 18 days, is the time that the beer will ſerve.

If wine gallons were to be converted into beer gallons,
Say 282 : 231 : : wine gallons : beer gallons.
Or 94 : 77 : : W. G. : B. G.

## ARTICLE VII.

### *Of the proportion of the Engliſh and French feet and pounds.*

Near the point of the face D of the Callipers are two tables ſhewing the proportion between the pound weights of *London* and *Paris*, and alſo between the lengths of the foot meaſure of *England* and *France*. Theſe are according to the accurate ſtandards ſettled between the Royal Societies of *London* and *Paris* about the year 1743.

### THE TABLES.

| Eng. ℔. | | F. ℔. | Eng. F$^{t}$. | | Fr. F$^{t}$. |
|---|---|---|---|---|---|
| 1,08 | = | 1,00 | 114 | = | 107 |
| 1,00 | = | 0,926 | 1,000 | = | 0,9386 |
| 108 | = | 100. | 1,0654 | = | 1,0000 |

### THE USE.

Example I. *Suppoſe a crew of 54 Engliſh ſailors were to attack a French fort, and carry off 6 pieces of braſs cannon weighing one with another 980 ℔ French: How much would each John's ſhare come to, ſuppoſing they could ſell the cannon at 8 l. a hundred weight Engliſh?*

℔ F. ℔ E. ℔ F.
Now 100 : 108 : : 980×6 : 6350,4, ℔ Engl.
℔ £. £.
And 112 : 8 : : 6350,4 : 453,6 £. ſterling.
M. £. M.
Then 54 : 453,6 : : 1 : 8,4 £.
So that the ſhare of each will be 8 guineas.

Example II. *How many Engliſh yards are equal to 180 French toiſes or fathoms?*

Now 1 : 1,0654 : : 180 : 191,672 Eng. Fa.
Then 180 French Fathoms are equal to about 383 yards 1 foot.

## ARTICLE VIII.

### *Factors uſeful in circular and ſpherical figures.*

Near the point of the Callipers on the face A is a table containing four rules of the circle and ſphere.

The

## The Table.

| | | | |
|---|---|---|---|
| Diam. | ×3,1416 | =circumf. | of a circle. |
| Sq. Diam. | ×0,7854 | =area | |
| Sq. Diam. | ×3,1416 | =ſurface | of a ſphere. |
| Cube Diam. | ×0,5236 | =ſolidity | |

## THE USE.

Example I. *What is the circumference of a circle whoſe diameter is 12 inches?*

Then (3,1416×12 =) 37,6992 is the circumfer.

Example II. *What is the area of a circle whoſe diameter is 12 inches?*

Now the ſquare of 12 is 144.
Then (0,7854×144 =) 113,0976 is the area.

Example III. *What is the ſuperficies of a ſphere whoſe diameter is 12 inches?*

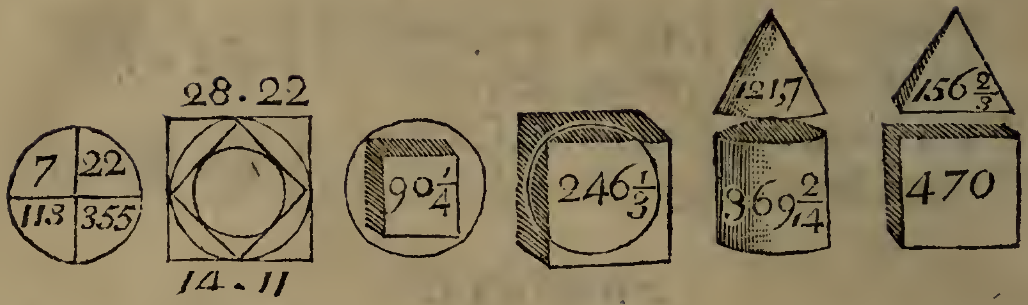
Now the ſquare of 12 is 144.
Then (3,1416×144=) 452,3904 the ſuperficies of the ſphere.

Example IV. *Required the ſolidity of a ſphere whoſe diameter is 12 inches?*

Now the cube of 12 is 1728.
Then (0,5236×1728=) 904,7808 is the ſolidity.

Upon the circular heads of Callipers are uſually placed certain mathematical figures with numbers ſet to them; which figures and their numbers may be placed near the points of the Callipers here deſcribed, the circular head being appropriated for another uſe.

*The figures are theſe.*



1 2 3 4 5 6

The numbers in figure 1, are uſeful for finding the circumference of a circle by knowing its diameter; or to find the diameter by knowing the circumference. Thus

Say As 7 : 22 : : any given diam: its circum.
And As 22 : 7 : : any given circum: its diam.
Or As 113 : 355 : : any given diam: its circum.
And As 355 : 113 : : any given circum: its diam.

Fig. 2. There is a circle inſcribed in a ſquare; a ſquare within that circle, and a circle within the inner ſquare: To this figure are ſet the numbers 28. 22. 14. 11. Theſe numbers ſignify, that if the area of the outward ſquare is 28, the area of the inſcribed circle is 22; the area of the ſquare inſcribed in that circle is 14, and the area of its inſcribed circle is 11.

### THE USE.

Example. *What is the area of a circle whoſe diameter is* 12?

Now the ſquare of 12 is 144.
Then As 28 : 22 : : 144 : 113,1 the area.
Or As 14 : 11 : : 144 : 113,1.

It may be obſerved, that the ſquares are in proportion to one another as 2 to 1; and the two circles are alſo in the ſame proportion.

*Figure* 3. Repreſents a cube inſcribed in a ſphere; the number $90\frac{1}{4}$ fixed to it ſhews, that a cube of iron, inſcribed in a ſphere of 12 inches in diameter, weighs $90\frac{1}{4}$ pounds weight.

*Figure* 4. Is to expreſs a ſphere inſcribed in a cube: Now this figure with its number $246\frac{1}{4}$ is to ſhew the weight in pounds of an iron ſphere of 12 inches diameter; or of a ſphere inſcribed in a cube whoſe ſide is 12 inches.

*Figure* 5. Repreſents a cylinder and cone, whoſe diameters and heights are each one foot: To the cylinder is annexed the number $369\frac{3}{14}$ ſhewing the weight in pounds of an iron cylinder of 12 inches diameter and 12 inches in height: And the number $121\frac{7}{100}$ joined to the cone ſhews that an iron cone the diameter of whoſe baſe is 12 inches, and the height 12 inches, weighs $121\frac{7}{100}$ pounds.

*Figure* 6. Shews that an iron cube, whoſe ſide is 12 inches, weighs 470 pounds; and that a ſquare pyramid of iron, whoſe baſe is a ſquare foot, and its height 12 inches, weighs $156\frac{2}{5}$ pounds.

THESE numbers which have hitherto been fixed to the four laſt figures are not ſtrictly true.

FOR by experiment an iron ſhot of four inches diameter weighs 9 pounds.

AND the weights of ſpheres being to one another as the cubes of their diameters:

THEREFORE 64 $(=4\times4\times4)$ : 9 : : 1728 $(=12\times12\times12)$ : 243 pounds, for the weight of a ſphere of iron which is 12 inches in diameter: Conſequently the number 243 ſhould be uſed inſtead of $246\frac{1}{4}$ in the 4th figure.

AGAIN. The ſolidity of a cube inſcribed in a ſphere of 12 inches in diameter, is 332,55 cubic inches.

AND the weights of bodies of a like matter being in the proportion of their ſolidities.

THEREFORE, As 904,7808 : 243 : : 332,55 : 89,315 pounds.

CONSEQUENTLY the number $90\frac{1}{4}$ uſed at figure 3, ſhould be $89\frac{1}{3}$.

Here 904,7808 is the ſolidity of a ſphere of 12 inches diameter.

At figure 5. the weight of the iron cylinder ſhould be 364,5 inſtead of $369\frac{3}{14}$, and the weight of the cone ſhould be 121,5.

For the ſolidity of a cylinder of 12 inches diameter, and 12 inches high, is 1357,1712 cubic inches.

Then 904,7808 : 243 : : 1357,1712 : 364,5 pounds.

And cylinders and cones having equal baſes and heights are in proportion as 3 to 1.

Therefore the $\frac{1}{3}$ of 364,5, or 121,5 pounds is the weight of the cone.

The numbers at figure 6 annexed to the cube ſhould be 464 pounds.

And that fixed to the pyramid ſhould be $154\frac{2}{3}$ pounds.

For the cube inches in a foot cube are 1728.

Then 904,7808 : 243 : : 1728 : 464.

And a pyramid is $\frac{1}{3}$ of a cube, the baſes and height being equal.

Therefore the $\frac{1}{3}$ of 464 is $154\frac{2}{3}$ pounds for the weight of the pyramid.

Although it is uſually reckoned that a four inch iron ſhot weighs nine pounds; and from thence it is deduced that the cubic foot weighs 464 pounds; yet by the table of ſpecific gravity on the callipers, which is framed from the moſt accurate experiments, a cubic foot of caſt iron weighs almoſt 446 pounds; which is 18 pound leſs than the weight derived from the 4 inch ſhot, and 24 pound leſs than that heretofore graved on the callipers; therefore all the weights found from the ſaid 4 inch ſhot, ſhould be diminiſhed in the proportion of 464 to 446.

For the numbers at figures 3, 4, 5, 6.

As 464 : 446 : : 89,315 : 85,85.
As 464 : 446 : : 243 : 233,5.
As 464 : 446 : : 364,5 : 350,3.

So $85\frac{4}{5}$ ℔ is the weight of an iron cube inſcribed in a ſphere of 12 inches in diameter.

And $233\frac{1}{2}$ ℔ is the weight of an iron ſphere of 12 inches diameter.

Also $350\frac{1}{3}$ ℔ is the weight of an iron cylinder of a foot in diameter and height.

And $116\frac{2}{3}$ ℔ is the weight of an iron cone of a foot in diameter and height.

Again 446 ℔ is the weight of a cubic foot of iron.

And $148\frac{2}{3}$ ℔ is the weight of an iron pyramid, having its baſe a ſquare foot, and its height equal to 12 inches.

## ARTICLE IX.

### *Of the ſpecific gravities and weights of bodies.*

On the leg B of the callipers is a table ſhewing the weights of a cubic inch or foot of various bodies in pounds averdupoiſe. To the table here annexed is joined the ſpecific gravities of thoſe bodies, which are omitted on the callipers for want of room.

*A*

## *A Table ſhewing the weights of bodies and their ſpecific gravities.*

| Bodies. | Weights. | Spe. Gravity. |
|---|---|---|
| Fine Gold. Inch | 0,710359 | 19,640 |
| Standard Gold. Inch | 0,706018 | 19,520 |
| Quickſilver. Inch. | 0,497657 | 13,762 |
| Lead. Foot | 707,0458 | 11,313 |
| Inch | 0,409170 | |
| Fine Silver. Inch | 0,401150 | 11,091 |
| Standard Silver. Inch | 0,384440 | 10,629 |
| Copper Foot | 548,0628 | 8,769 |
| Inch | 0,317166 | |
| Braſs. F. | 506,2746 | 8,104 |
| Steel. F. | 490,6241 | 7,850 |
| Bar Iron. F. | 485,2500 | 7,764 |
| Block Tin. F. | 452,3731 | 7,238 |
| Caſt Iron. F. | 445,9363 | 7,135 |
| White Marble. F. | 168,8757 | 2,702 |
| Glaſs. F. | 162,4994 | 2,600 |
| Stone Flint. F. | 161,3745 | 2,582 |
| Portland. F. | 160,6245 | 2,570 |
| Free. F. | 158,2485 | 2,352 |
| Brick. F. | 125,0000 | 2,000 |
| Brimſtone. F. | 112,5000 | 1,800 |
| Clay. F. | 112,0000 | 1,792 |
| River Sand. F. | 110,0000 | 1,760 |
| Sea Water. F. | 64,3732 | 1,030 |
| Rain Water Cubic F. | 62,5000 | 1,000 |
| Cubic Inch | 0,036169 | |
| Cylindric F. | 49,080000 | |
| Cylindric Inch | 0,028403 | |
| Port Wine. F. | 61,8000 | 0,988 |
| Brandy. F. | 58,0000 | 0,928 |
| Olive Oil. F. | 57,0624 | 0,913 |
| Dry Oak. F. | 57,1875 | 0,915 |
| Lime. F. | 52,0000 | 0,832 |
| Elm and Aſh. F. | 50,0000 | 0,800 |
| Wheat. F. | 50,0000 | 0,800 |
| Yellow Fir. F. | 41,0625 | 0,657 |
| White Deal. F. | 35,5624 | 0,569 |
| Gun Powder F. | 69,1200 | 1,106 |
| In. | 0,0400 | |

In the foregoing table is contained ſuch bodies as practical engineers and others may have occaſion to know their reſpective weights; there are indeed a great number of other bodies whoſe ſpecific gravity have been determined by ingenious men: But thoſe only which were apprehended to be the moſt uſeful were ſelected for this ſubject.

Every one will readily conceive how the column of weights may be obtained, namely by procuring maſſes of a cubic inch or foot of the ſolids, and carefully weighing them in nice ſcales to the ſmalleſt degree of averdupoiſe weight: And for the fluids, their weights may be determined by having cubical or cylindrical veſſels made to hold a known quantity of cubical inches, and in them to weigh the fluids.

The ſpecific gravity of a body being the relation which that body has to ſome other body fixed upon as a ſtandard to compare by; and rain water being found to be alike, or very nearly ſo, in all places; and therefore choſen by philoſophers as the proper ſtandard; conſequently by the word ſpecific gravity of a body is meant no more, than that it is ſo many times heavier or lighter than water, when compared together in equal balks.

Thus fine ſilver is ſomething more than 11; that is, a maſs of fine ſilver will weigh ſomething above eleven times the weight of an equal maſs of water: And, ſo a common brick weighs twice as much as the rain water that would fill a mould fitted to the brick.

Now the weights of equal maſſes of ſeveral bodies being determined, their ſpecific gravities may be readily found, they being in the ſame proportion to one another as their weights: And as the compariſon is made to rain water, of which, by repeated experiments, it has been found that a cubic foot weighed $62\frac{1}{2}$ pounds averdupoiſe; therefore *dividing the weight of a cubic foot of any body, by* $62\frac{1}{2}$; *the quotient will be* *the ſpecific gravity of that body*, relative to rain water whoſe ſpecific gravity is repreſented by unity.

THE difficulty of procuring maſſes of metals and other bodies in all parts homogenous, and of having both them and the veſſels of capacity conſtructed to a mathematical exactneſs, has rendered this method of eſtimating the ſpecific gravities from the weights of equal bulks, liable to exception: And therefore another method has been contrived to come at theſe ſpecific gravities, hydroſtatically.

IT is a well known thing that any body will weigh leſs when it is immerſed in water than when it is weighed in the open air; and from a very little reflection, it will be ſeen that the difference between the weights of any body when weighed in air and in water, will be equal to the weight of ſo much water as is equal in bulk to the body immerſed: But the difference between the weights of a body in air and in water, will ſhew the weight of a bulk of water equal to the body ſo weighed: *Therefore to find the ſpecific gravity of any body, find its weight in air and in rain water, and take the difference of thoſe weights; then the weight in air divided by that difference, will give the ſpecific gravity required.*

IF the ſolid whoſe ſpecific gravity is wanted, be lighter than water, ſo that it cannot ſink by its own weight, let it be joined to another ſo weighty that the compound may ſink: But firſt let the loſs of weight which the heavy body alone ſuſtains in water be found as before; and then let the loſs of weight which the compound body ſuſtains be diſcovered; from which take the loſs of weight of the heavier, and the remainder is the loſs of weight ſuſtained by the lighter; by which dividing the weight in air of the lighter body, and the quotient will ſhew the ſpecific gravity.

WHEN the ſpecific gravity of fluids are to be compared to each other; take a ſolid of any matter and ſhape, ſuppoſe a glaſs ball, hung by a horſe hair, and immerſe this ſolid in each fluid, and find the loſs of weight of the ſolid in each fluid, the weight of the body in air being firſt known; then will theſe loſſes expreſs the ſpecific gravities of thoſe fluids: For ſince the loſs of weight in each liquor is equal to the weight of as much of the liquor as is equal in bulk to the body weighed; therefore by taking the loſſes of weight ſuſtained by the ſame body in the ſeveral liquors, the abſolute weights are obtained of ſuch portions as are equal in bulk, and conſequently the ſpecific gravities of thoſe liquors.

In this method of finding the ſpecific gravity of ſolids, it is not neceſſary that they ſhould be reduced to any regular ſhape; neither is there wanted a veſſel of a known figure and capacity to contain the fluids; and conſequently the ſpecific gravities of bodies, whether ſolids or fluids, may be very eaſily come at: But from the ſpecific gravities to find the abſolute weights of any aſſigned maſs of ſeveral bodies, there muſt be another experiment made, which is to find the loſs of weight in water, of a body of a known magnitude; ſuppoſe of a cylinder of a homogenous metal, the ſolidity of that cylinder being moſt accurately calculated; then will the abſolute weight of an equal maſs of water be known; and conſequently the weight of a cubit foot of water may be accurately obtained, *from whence the abſolute weight of a cubic foot of any other body whoſe ſpecific gravity is known, may be found by multiplying the ſpecific gravity of that body by the weight of a cubic foot of water.*

### SOME USES OF THE TABLE.

The weights of bodies anſwering to a given ſolidity are of a twofold uſe.

First, *To find the weight of a body of a given dimenſions, or ſolidity.*

Secondly, *To find the ſolidity of a body by knowing its weight.*

Exam. I. *What is the weight of a block of marble 7 feet long, 3 feet broad, and 2 feet thick?*

Now 7×3×2=42 feet for the ſolidity.

A cubic foot of marble weigh 168,8757 pounds.

Then 168,8757×42 gives 7092,7794 pounds.

C qrs. ℔

Or, 63 : 1 : $8\frac{3}{4}$ is the weight of that marble.

Exam. II. *What is the weight of a 13 inch iron bomb ſhell, the metal being two inches thick on a mean?*

Here the ſolidity of two ſpheres muſt be found, one of 13 inches diameter, and the other of 9 inches diameter; then their difference being taken will give the ſolidity of the ſhell.

Now the cube of 13 is 2197.

And the cube of 9 is 729,

Also 2197×0,5236 gives 1150,3492 ſolidity.

And 729×0,5236 gives 381,7044 ſolidity.

Their difference is 768,6448 cubic inches.

And 768,6448 divided by 1728 gives 0,4448 parts of a cubic foot.

Now a cubic foot of caſt iron weighs 445,9363 pounds.

Then 445,9363×0,4448 gives 198,363 pounds for the weight of the ſhell.

Exam. III. *How many pigs each of 12 inches long, 6 wide and 4 thick, may be caſt out of 10 ton of melted lead?*

Now 10 ton =10×20=200 C. wt.

And 112×200=22400 pounds in 10 ton.

By the table, 707,0458 pound makes a cubic foot of lead.

And 22400 divided by 707,0458, gives 31,681 cubic feet, which the 10 ton will make.

Now the ſolidity of each pig is $\frac{1}{6}$ of a foot.

THEREFORE 31,681 feet ſolid will make 190 pigs.

FROM ſeveral experiments it appears that middling ſized men, or thoſe between 5 feet 6 inches and 5 feet 9 inches in height, weigh about 150 pounds, and are in bulk equal to about $2\frac{3}{4}$ ſolid feet; and the ſmall ſized men, or thoſe between 5 feet 3 inches, and 5 feet 6 inches in height, weigh about 135 pounds, and are in bulk equal to about $2\frac{1}{2}$ ſolid feet: And from thoſe expriments it alſo appears, that moſt men are ſpecifically lighter than common water, and much more ſo than ſea water. Conſequently could perſons who fall into water, have preſence of mind enough to avoid the fright uſual on ſuch occaſions, many might be preſerved from drowning: And a very ſmall piece of wood, ſuch as an oar, would buoy a man above water while he had ſpirits to keep his hold.

A GENTLEMAN who had been on board of a Malteſe ſhip of war, obſerved hanging to the tafarel, a block of wood almoſt like a buoy, and ſo ballanced that one end ſwam upright, carrying a little flagſtaff with a ſmall vane; the perſon who was on duty on the poop had orders to cut the rope by which the buoy hung, upon any cry of a perſon's falling over board; and as the block would be in the ſhip's wake by the time the perſon floated therein, he was ſure of having ſomething at hand to ſuſtain him, till the boat could come to his aſſiſtance; and ſhould that take ſo long time to do, as that the diſtance from the ſhip to the man rendered him inviſible, yet the boat would have a mark to row towards, ſhewn them by the vane.

EXAM. IV. *How many ſpars of white fir, each of* 20 *feet long and a foot ſquare, are to be laſhed together, till the raft is ſufficient to float, in common water,* 100 *barrels of gunpowder conducted by four middling ſized men, ſo as to keep the barrels three inches clear of the water?*

A barrel of gunpowder, barrel and all, weighs about 120 ℔.

So 100 barrels will weigh 12000 ℔.

And 4 men, at 150 ℔ each, weigh 600 ℔.

So that the raft muſt ſuſtain a weight of 12600 ℔.

Now the deal will of it ſelf ſink in the water, until the weight of the water diſplaced is equal to the weight of the wood.

In each ſpar there is 20 feet of timber.

A cubic foot of white deal weighs 35,5624 pounds.

So 35,5624×20=711,248 ℔. the weight of one ſpar.

And is alſo equal to the weight of the water diſplaced.

A cubic foot of common water weighs 62,5 ℔.

Then 62,5 : 1 : : 711,248 : 11,38 the number of cubic feet which each ſpar will have immerſed by its own weight.

As the barrels are to be 3 inches clear of the water, therefore the ſpar muſt be ſunk 9 inches; and conſequently 15 feet ſolid of each ſpar muſt be immerſed:

Then 15—11,38=3,62 the additional cubic feet of water to be diſplaced by each ſpar, by its incumbent weight.

And 1 : 3,62 : : 62,5 : 226,25℔. the weight which each ſpar is to ſuſtain.

Then 226,25 : 12600 : : 1 : 55,6, &c.

Consequently 56 ſuch ſpars laſhed together will make a float ſufficient for to ſuſtain the given weight in the manner propoſed.

## ARTICLE X.

### *Of the quantity of powder uſed in firing of cannon.*

On the circular head of the callipers, on the face d is a table contained between five concentric ſegments of circular rings; the inner one markt Guns, ſhews the nature of the gun, or the weight of ball it carries: The two next rings contain the quantity of powder uſed for proof and ſervice to braſs guns; and the two outermoſt rings ſhew the quantity for proof and ſervice, uſed in iron cannon.

The numbers in this table expreſs the Engliſh uſage, which for the moſt part, allows the weight of the ſhot for proof, half its weight in ſervice, and one fourth of its weight of ſhot for ſalutes.

The French allowance of powder, for the charge of the piece for ſervice, uſed to be two thirds of the weight of the ſhot; twice as much for proof, and one fourth of the weight of ſhot for ſalutes.

### THE TABLE.

| Nature of guns | Braſs | | Iron | | Salutes | Scaling |
|---|---|---|---|---|---|---|
| | Proof | Service | Proof | Service | | |
| Pounders | ℔. oz. | ℔. oz. | ℔. oz | ℔. oz. | ℔. oz. | ℔. oz. |
| 1 | 1.0 | 0.8 | 1.0 | 0.8 | 0.8 | 0 .1½ |
| 1½ | 1.8 | 0.12 | 1.8 | 0.12 | 0.12 | 0 .2 |
| 2 | 2.0 | 1.0 | 2.0 | 1.8 | 1.0 | 0 .3 |
| 3 | 3.0 | 1.8 | 3.0 | 1.8 | 1.8 | 0 .4 |
| 4 | 4.0 | 2.0 | 4.0 | 2.0 | 2.0 | 0 .6 |
| 5¼ | 5.4 | 2.10 | 5.4 | 2.10 | 2.10 | 0 .8 |
| 6 | 6.0 | 3.0 | 6.0 | 3.0 | 3.0 | 0 .8 |
| 8 | 8.0 | 4.0 | 8.0 | 4.0 | 3.12 | 0 .10 |
| 9 | 9.0 | 4.8 | 9.0 | 4.8 | 4.0 | 0 .12 |
| 12 | 12.0 | 6.0 | 12.0 | 6.0 | 4.12 | 1 .0 |
| 18 | 18.0 | 9.0 | 15.0 | 9.0 | 6.0 | 1 .8 |
| 24 | 21.0 | 12.0 | 18.0 | 11.0 | 7.0 | 2 .0 |
| 26 | 22.0 | 13.0 | 19.0 | 12.0 | 7.12 | 2 .4 |
| 32 | 26.12 | 16.0 | 21.8 | 14.0 | 9.4 | 2 .12 |
| 36 | 28.0 | 18.0 | 22.0 | 15.0 | 10.0 | 3 .0 |
| 42 | 31.8 | 21.0 | 25.0 | 17.0 | 11.4 | 3 .4 |

GUNS carrying ſhot of the weight 1 lb. 1½. lb. 2 lb. 4 lb. 5¼ lb. 8 lb. 26 lb. 36 lb. are now out of uſe in the Britiſh navy.

THE uſe of this table is obvious: For ſeek the name of the gun in the inner ring, and the weights of powder for proof and ſervice will be found between the ſame two ſtrait lines, like radii; and in one of the other rings, according as it is tituled at the end.

THUS to a braſs 9 pounder, there is allowed 9lb. of powder for to prove, or try the goodneſs of the gun when it is firſt caſt; and 4 lb. 8 oz. of powder for each charge in common ſervice: But an iron 9 pounder has 9 lb. for proof, and 6 lb. for ſervice.

WHEN cannon are proved they are uſually loaded with two ſhot.

ON ſhip board, after there are five or ſix rounds fired on warm ſervice, the allowance of powder is to be proportionally leſſened each time the gun is loaded, until the charge is reduced to one third of the weight of the ſhot: And the guns as they grow warm in firing, are not to be wetted leſt the gun be in danger of ſplitting by checking the metal with cold water.

THE ingenious Mr. *Robins*, from ſome hints he gathered from a manuſcript lent him by the Right Honourable Lord *Anſon*, adviſes to leſſen conſiderably the common charges allowed to cannon in ſervice: For from thoſe papers it appeared that in ſervice, where 24 pounders have been uſed to batter in breach, the charge was only 8 pounds of powder: Indeed the velocity of the ball could not be quite ſo great with 8 pounds of powder as with 12, and conſequently the ſhot would not be drove ſo far into the rampart, and the breach not made altogether ſo ſoon; notwithſtanding which, the advantages attending the ſmaller charges, greatly overbalanced the difference of a few hours in making a ſufficient breach.

IN ſea ſervice it would perhaps be found of greater uſe to begin with one third of the weight of ſhot in powder, and to diminiſh that to one fourth or one fifth as the gun waxed warm; for by ſome experiments it has appeared, that ſuch ſmall charges of powder has produced greater ravage in timber, than has been found with the uſual charges: From whence it may be reaſonably concluded, that if a ſhot has juſt force enough to go through one ſide of a ſhip, there will be a greater quantity of ſplinters rent out of the plank, and conſequently do more miſchief, than if the ſhot went with a velocity ſufficient to drive it through both ſides of the ſhip.

## ARTICLE XI.

### *Of the number of ſhot or ſhells in a finiſhed pile.*

Iron ſhot and ſhells are uſually piled up by horizontal courſes into a pyramidal form, the baſe being either an equilateral triangle, or a ſquare; or a rectangle; in the triangle and ſquare, the pile finiſhes in a ſingle ball; but in the rectangle, the finiſhing is a ſingle row of balls.

In the triangular and ſquare piles, the number of horizontal rows, or the number counted on one of the angles from the bottom to the top, is always equal to the number counted on one ſide, in the bottom row.

In triangular piles, each horizontal courſe is a triangular number, produced by taking the ſucceſſive ſums of the numbers 1 and 2; 1, 2 and 3; 1, 2, 3 and 4; 1, 2, 3, 4 and 5, &c. Thus.

Numbers in order 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 . 10 . 11, &c.
Triangular numb. 1 . 3 . 6 . 10 . 15 . 21 . 28 . 36 . 45 . 55 . 66, &c.

And the number of ſhot in a triangular pile is the ſum of all the triangular numbers taken as far, or to as many terms, as the number in one ſide of the bottom courſe.

### *A rule to find the number of ſhot in a triangular pile.*

Count the number in the bottom row, and multiply that number more two by that number more one: Then the product multiplied by one ſixth of the ſaid number, the product will be the ſum of all the ſhot in the pile.

EXAM. I. *How many ſhot are in a finiſhed triangular pile, in one ſide of whoſe bottom courſe are 20 ſhot?*

Now the number more two is 22; and the number more one is 21.

AND 22×21 gives 462.

THEN $462\times\frac{20}{6}=1540$, the number of ſhot in that pile.

EXAM. II. *Required the number of ſhot in a finiſhed pile; there being in one ſide of the triangular baſe* 40 *ſhot?*

HERE the number more two is 42; and the number more one is 41.

AND 42×41 gives 1722.

THEN $1722\times\frac{40}{6}=11480$ ſhot in that pile.

IN ſquare piles, each horizontal courſe is a ſquare number, produced by taking the ſquare of the number in its ſide.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Number in the ſide | 1. | 2. | 3. | 4. | 5. | 6. | 7. | 8. | 9. | 10, | &c. |
| Squares, or horiz. courſes | 1. | 4. | 9. | 16. | 25. | 36. | 49. | 64. | 81. | 100, | &c. |

AND the number of ſhot in a ſquare pile is the ſum of all the ſquares, taken from one, as far as the number in the ſides of the bottom courſe.

*A rule to find the number of ſhot in a ſquare pile.*

COUNT the number in one ſide of the bottom courſe; to that humber add one, and to its double add one; multiply the two ſums together; then their product being multiplied by one ſixth of the ſaid number, the product will give the number of ſhot contained in that pile.

Exam. III. *How many ſhot are in a ſquare finiſhed pile, one ſide of its baſe containing 20 ſhot?*

Here the number is 20.

The number more one is 21; and its double, more one is 41.

The product of theſe numbers is 861 ($=21\times41$)

Then $861\times\frac{20}{6}=2870$, the number of ſhot in that pile.

Exam. IV. *Required the number of ſhot in a ſquare finiſhed pile, one ſide of the lower courſe, or tier, having 40 ſhot in it?*

Here the number counted is 40.

That number more one is 41; its double, more one is 81.

And $81\times41=3321$ the product.

Then $3321\times\frac{40}{6}=22140$ the number in that pile.

From theſe examples it may be obſerved, that where room is wanted, 'tis moſt convenient to have the ſhot ſtowed in triangular piles: For on the equilateral triangle, which is leſs than half the area of a ſquare on one of its ſides, there can be piled a greater number than half of what can be raiſed on the ſquare: Indeed the height of a ſquare pile is ſomewhat leſs than a triangular one, as a ſhot will ſink lower in the ſpace between 4 others, than in the ſpace between 3 others, all the ſhot being of equal diameter; they being ſo reckoned in every pile.

In rectangular piles, each horizontal courſe is a rectangle, the upper one being one row of balls: Now every ſuch oblong pile may be conſidered as conſiſting of two parts, one a ſquare pyramid, and the other a triangular priſm.

*To find the number of ſhot in a rectangular pile.*

1ſt. Take the difference between the number in length and breadth in the bottom courſe.

2d. Multiply the number in breadth, more one, by half the breadth; the product multiplied by the ſaid difference, will give the number in the priſmatic pile.

3d. Upon the ſquare of the breadth, find (by the laſt rule) the number in a pyramidal pile.

4th. Then the ſum of theſe two piles will ſhew the number in the rectangular pile.

N. B. The number of horizontal courſes, or rows, is equal to the number in breadth at bottom: And the number leſs one, in the top row, is the difference between the number in length and breadth at bottom.

Exam. V. *How many ſhot are in a finiſhed pile of* 20 *courſes, the number in the top row being* 40?

Here 39 is the difference between the length and breadth.

And 20 is the breadth.

Now $20+1=21$; and $2\times20+1=41$.

Then $21\times41\times\frac{20}{6}=2870$, are the ſhot in the pyramidal pile.

Again. The breadth more one is 21; and 10 is the half breadth.

And $21\times10=210$.

Then $210\times39=8190$, are the ſhot in the priſmatic pile.

Consequently the ſum of 2870 and 8190, or 11060 ſhot will be the number contained in that rectangular pile.

If any of theſe piles are broken, by having the upper part taken off, and the remaining number of ſhot are required; it may be obtained by computing what the whole finiſhed pile would contain; and alſo what the pile wanting, or taken away contained; for then their difference will ſhew the number remaining.

The foregoing rules are thus expreſſed on the Callipers.

Number of ſhot or ſhells in a pile.

Let

$$\text{Let }\left.\begin{array}{l} n=\text{N}^{\circ}\text{ in an angular row} \\ m=\text{N}^{\circ}\text{ leſs one in the top row} \end{array}\right\}\text{of a Pile.}$$

$$\left.\begin{array}{l} \text{Then }\overline{n+2}\times\overline{n+1}\times\frac{n}{6}=\text{N}^{\circ}\text{ in a }\triangle \\ \text{And }\overline{n+1}\times\overline{2n+1}\times\frac{n}{6}=\text{N}^{\circ}\text{ in a }\square \\ \text{Also }\overline{\overline{2n+1}+3m}\times\overline{n+1}\times\frac{n}{6}=\text{N}^{\circ}\text{ in a }\boxed{\quad} \end{array}\right\}\text{Pile.}$$

In Examples I & III. The letter *n* ſtands for 20.
And Examples II & IV. The letter *n* ſtands for 40.
In Example V. The letter *n* ſtands for 20.
And the letter *m* ſtands for 39.

Then $2n+1=2\times20+1=41.$
And $3m=117.$
So $\overline{2n+1}+3m=158.$
Also $n+1=21.$
And $\frac{n}{6}=\frac{20}{6}$

Then $\overline{\overline{2n+1}+3m}\times\overline{n+1}\times\frac{n}{6}=158\times21\times\frac{20}{6}=11060.$

## ARTICLE XII.

### *Concerning the fall of heavy bodies.*

When heavy bodies are ſuffered to fall, it is well known they fall in lines perpendicular to the ſurface of the earth.

The force with which any body in motion ſtrikes an obſtacle, depends on the weight of that body, and on the velocity or ſwiftneſs with which it moves.

Thus a man by throwing, with the ſame ſtrength, a pound of iron and a pound of cork, will hit a much harder ſtroke with the iron than with the cork.

Also a man and a boy each throwing a pound of iron againſt the ſame object, the ſtroke given by the man will be ſtronger than that given by the boy, on account of the man's weight flying the ſwifteſt.

The ſame heavy body by falling from different heights, will ſtrike blows of different ſtrength, that being the ſtrongeſt where the height is greateſt. Conſequently heavy bodies by falling acquire velocities greater and greater according to the length of their fall.

The three following propoſitions in falling bodies have been proved many ways.

1ſt. *That the velocities acquired are directly proportional to the times.*

2d. *That the ſpaces fallen through are as the ſquares of the times, or as the ſquares of the velocities.*

3d. *That a body moving uniformly with the velocity obtained by falling through any height, will fall twice as far in the ſame time it was paſſing through that height.*

Experiments ſhew that heavy bodies fall about 16 feet in one ſecond of time: Conſequently at the end of the firſt ſecond of time, a falling body has acquired a velocity that would carry it down 32 feet in the next ſecond of time.

Then from the foregoing three propoſitions may be derived the following rules.

*1ſt.* That the ſquare root of the feet in the ſpace fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall.

*2d.* That the ſquare root of the feet in the ſpace fallen through, will ever be equal to four times the number of ſeconds of time the body has been falling.

*3d.* And that four times the number of ſeconds of time in which the body has been falling, is equal to one eighth of the velocity in feet per ſecond, acquired at the end of the fall.

From theſe three rules moſt of the queſtions relative to the fall of bodies may be readily ſolved.

As theſe rules cannot, for want of room, be put in words at length on the callipers, they are, on the face A of one of the legs, expreſſed in an algebraic manner. Thus,

### FALL OF BODIES.

LET $S=$ſpace run in feet.
$T=$time in ſeconds.
$V=$velocity in feet per ſecond.

THEN $\sqrt{S}=4T=\frac{1}{8}V$.

BODIES fall 16 feet in 1ſt ſec.

*Note.* The character $\sqrt{}$, ſignifies the ſquare root of the letter joined to it.

### SOME USES.

EXAM. I. *How many feet will a bullet fall in 5 ſeconds of time?*

HERE the time $T=5$;
THEN $4T$, makes $4\times5=20$.
NOW $\sqrt{S}=(4T=)\ 20$.
AND $S=(20\times20=)\ 400$.

EXAM. II. *From what height muſt a bullet fall to acquire a velocity of 160 feet per ſecond?*

THE rule is $\sqrt{S}=\frac{1}{8}V$.
HERE $V$ is 160 feet.
AND $\frac{1}{8}V=(\frac{160}{8}=)\ 200$.
THEN $S=(20\times200=)\ 400$ feet.

EXAM. III. *How long muſt a bullet be in falling to acquire a velocity of 160 feet per ſecond.*

THE rule is $4T=\frac{1}{8}V$.
HERE $V=160$ feet.
AND $\frac{1}{8}V=(\frac{160}{8}=)\ 20$.
SO $4T=20$.
THEN $T=(\frac{20}{4}=)\ 5$ ſeconds of time.

EXAM. IV. *How many ſeconds will it require for a heavy body to fall through a ſpace equal to 3375 yards?*

THE rule is $4T = \sqrt{S}$.

THEREFORE $T = \frac{1}{4}\sqrt{S}$.

HERE $S = 3375$ yards, or 10125 feet.

AND the ſquare root of 10125 is 100,6.

THEN 100,6 divided by 4 gives 25,15.

So that it will require 25″. 9‴ of time for the body to fall through 3375 yards.

## ARTICLE XIII.

### *Rules for the raiſing of water.*

EXPERIMENTS have ſhewn, that taking horſes and men of a moderate ſtrength, one horſe will do as much work in raiſing of water, and ſuch like labour, as five men can.

IT has been alſo found, that one man in a minute, can raiſe a hogſhead of water 12 feet high upon a mean: For a ſtout man, well plied with ſtrong liquor, will raiſe a hogſhead of water 15 feet high in a minute: Now as the quantity of liquor equal to a hogſhead was raiſed to theſe heights only by way of experiment for a few minutes, ſuch numbers ought not to be eſteemed as the common labour of a man who is to work 4 or 5 hours on a ſtretch: But it may be reckoned, that of common labouring men, taken one with another, one of them will raiſe a hogſhead of water to 8 feet in height in one minute, and work at that rate for ſome hours.

IT is quite indifferent in what manner the man is ſuppoſed to apply his force; whether by carrying the water in manageable parcels up a ſtair-caſe, or raiſing it by means of ſome machine: For the advantages gained by uſing of engines ariſes chiefly from the eaſe with which the power can be applied.

On the face A of the callipers, are the rules, thus denoted.

### To raiſe water.

The power $=P$ men.
Or to $\frac{1}{5}P$ horſes
Can raiſe to $8P$ feet high $=F$.
The quantity $H$, hhds. in $T$ min.
Or $G$, gallons in $60\,T$ ſeconds.
Or, $H\times F=P\times 8\times T$ minutes.
*N. B.* Cubic feet $\times 6{,}1277$ gives gall.

Here hogſheads are reckoned at 60 gallons, this eſtimate being nice enough for any computations on water engines.

### SOME USES.

Examp. I. *How many hogsheads can ſix horſes raiſe, by an engine, to 25 feet high in 3 hours?*

Now 6 horſes, at 5 men to a horſe, is equal to 30 men.
And the time 3 hours is equal to 180 minutes.
The height to be raiſed is 25 feet.
The general rule is $H\times F=P\times 8\times T$.
Here $F=25$; $P=30$; $T=180$.
And $H$ is required.

Then $H=\frac{P\times 8\times T}{F}$

Or $H=\left(\frac{30\times 8\times 180}{25}=\right)1728$ hogſheads.

*Hence this rule.* Multiply eight times the power by the time, the product divided by the height, gives the hogſheads.

Examp. II. *It is propoſed to throw out of a pond, by an engine, 432 tuns of water in 3 hours by ſix horſes; to what heighth can the water be raiſed?*

As 4 Hhds make one tun; ſo 432 tuns make 1728 Hhds.

And 3 hours, or 180 minutes is the time.

Also the power of ſix horſes, is equal to that of 30 men.

The general rule is $H \times F = P \times 8 \times T$.

Here $H = 1728$; $P = 30$; $T = 180$.

And $F$ is required.

$$\text{Then } F = \frac{P \times 8 \times T}{H}$$

$$\text{Or} \quad F = \left( \frac{30 \times 8 \times 180}{1728} = \right) \text{ 25 feet high.}$$

*Hence this rule.* Multiply eight times the power by the time; the product divided by the hogſheads, gives the height in feet.

Examp. III. *How long will it require ſix horſes to raiſe with an engine* 1728 *hogſheads of water to the height of* 25 *feet?*

Now the power 6 horſes, is equal to that of 30 men.

The hogſheads to be raiſed are 1728.

The height raiſed to is 25 feet.

The general rule is $H \times F = P \times 8 \times T$.

Here $H = 1728$; $F = 25$; $P = 30$.

And $T$ is required.

$$\text{Then } T = \frac{H \times F}{P \times 8}$$

$$\text{Or} \quad T = \left( \frac{1728 \times 25}{30 \times 8} = \right) \text{ 180 minutes, or 3 hours.}$$

*Hence this rule.* Multiply the hogſheads, by the height in feet; the product divided by 8 times, the power will give the time in minutes.

Examp. IV. *How many horſes will it require to work an engine, to raiſe* 1728 *hogſheads to the height of* 25 *feet, in* 3 *hours?*

Now the hogſheads to be raiſed are 1728.
The height to be raiſed is 25 feet.
The time to be done in is 3 hours, or 180 minutes.
The general rule is $h \times f = p \times 8 \times t$.
Here $h = 1728$; $f = 25$; $t = 180$.
And $p$ is required.

Then $p = \frac{h \times f}{8 \times t}$

Or $p = \left( \frac{1728 \times 25}{8 \times 180} = \right)$ 30 men, or 6 horſes.

*Hence this rule.* Multiply the number of hogſheads, by the height in feet; the product divided by 8 times the number of minutes, gives the number o men.

## ARTICLE XIV.

### *Of the ſhooting in cannon and mortars.*

It has been proved by many writers, that the flight of ſhot, or the track they deſcribe in the air, is a curve line called a parabola: But then they ſuppoſe that the reſiſtance made by the air is ſo inconſiderable as ſcarcely to affect the motion of heavy bodies.

Upon this ſuppoſition then, which is very far from being true; there have been collected the following obſervations and rules.

I. All bodies projected by any force; are urged with two motions, viz. one in the direction of the power exerted by the engine, and the other in a perpendicular direction to the earth, by the force of gravity; and the track or path deſcribed by the body with theſe two forces is a curve called the parabola.

II. The axis of the curve will be at right angles to the horizon; and the part in which the body deſcends will be alike to that in which it aſcended.

III. If the point to which the body arrives in its deſcent, be on the ſame level with the point from which it was projected, thoſe points are equally diſtant from the vertex, or higheſt point of the curve.

IV. If a body be projected oblique to the horizon, it will fall there again in the ſame obliquity, and with the ſame velocity it was projected withal.

V. The horizontal ranges of equal bodies, when projected with the ſame velocity, at different elevations, will be in proportion to one another; as the right ſines of twice the angles of elevation.

VI. Among equal bodies, projected with equal velocities, the heights to which they will riſe in the air, are in the ſame proportion to one another as the verſed ſines of twice the angles of elevation.

VII. When equal bodies are projected with equal velocities, the times of their continuance in the air will be in proportion to one another as the right ſines of the angles of elevation.

VIII. In the ſame piece, different charges of equally good gunpowder will produce velocities, nearly in the ſame proportion as the ſquare roots of the weights of the charges.

IX. If equal bodies be projected at the ſame elevation, but with different velocities, the horizontal ranges will be in proportion to one another, as the ſquares of the velocities given to the ſhot, or as the weights of the charges of powder nearly.

X. The greateſt horizontal range is double to the height from which the body ſhould fall to acquire that force or velocity which would project it to that horizontal range.

XI. The greateſt horizontal range, or diſtance to which a body can be thrown, will be obtained when it is projected at an angle of 45 degrees of elevation.

XII. The greateſt height to which a projected body can riſe, at an elevation of 45 degrees, is equal to one fourth part of its horizontal range.

XIII. To hit an object that lies above or below the horizon of the piece, the beſt elevation, is equal to the complement of half the angular diſtance between the object and the zenith.

XIV. At elevations equally diſtant from 45 degrees, both above and below it, the horizontal ranges will be equal.

XV. The time which a heavy body, projected at an elevation of 45 degrees, will continue in the air, before it arrives at the horizon, will be equal to the time that body would take to deſcend, by the force of gravity, through a ſpace equal to the horizontal range.

It has been found that a 24 pounder at an elevation of 45 degrees, and charged with 16 pound of powder, has ranged its ſhot upon the horizontal plane about 6750 yards.

Therefore 3375 is the impetus, or perpendicular ſpace which a 24 pounder muſt fall through to acquire ſuch a velocity, as, at an elevation of 45 degrees, would project or throw that ſhot on the horizon to the diſtance of 6750 yards.

Now a heavy body falling by the force of gravity through a ſpace equal to 3375 yards or 10125 feet, will, at the end of the fall, acquire a velocity of 804,8 or about 805 feet per ſecond (as ſhewn at Art. XII.)

And to fall through a ſpace of 805 feet, it would require 25″. 9‴ of time.

The chief of the above principles are ſhortly expreſſed on the face B of the callipers in the following manner.

## RULES FOR SHOOTING.

Hor. ranges, as right } ſines of twice } angles of
Heights, are as verſed } elevation.
Time in air, as right ſines of——

Impetus $=\frac{1}{2}$ } Hor. range, at 45 deg. of elevation.
Height $=\frac{1}{4}$ }

In aſcents or deſcents, for the beſt elevation.

Take the complement of $\frac{1}{2}$ the angular diſtance from object to zenith.

To apply theſe rules to the practice of ſhooting, it is to be underſtood that the gunner ſhould make an experiment with every gun he has the care of at ſome elevation, ſuppoſe at 45 degrees, and with the uſual charge of powder, and then knowing how far the piece has ranged the ſhot on the horizontal plane; he may apply the reſult of thoſe experiments to other elevations and quantities of powder.

Examp. I. *Suppoſe the greateſt horizontal range to be* 6750 *yards: How far will the ſame piece, and with an equal charge of powder, range a ſhot at an elevation of* 25 *degrees?*

With equal charges the horizontal ranges are as the right ſines of twice the angles of elevation.

Then, As radius, or the ſine of twice 45°
Is to the ſine of 50°, or the ſine of twice 25°,
So is the greateſt horizontal range 6750 yds
To the horizontal range required. 5170 yds.

That is, The extent on the line of ſines from 90° to 50°.

Will on the line of numbers reach from 6750 to 5170.

Examp. II. *The greateſt horizontal range of a* 24 *pounder being* 6750 *yards: To what height will that ſhot riſe at an elevation of* 25 *degrees?*

At an elevation of 45°, the ſhot will riſe $1687\frac{1}{2}$ yards, $=\frac{1}{4}$ of 6750.

And the heights are as the verſed ſines of twice the angles of elevation.

Then, As the verſed ſine of 90 degres, or of twice 45°.
Is to the verſed ſine of 50 degrees, or of twice 25°.
So is the height of an elevation of 45°, viz. $1687\frac{1}{2}$.
To the height at an elevation of 25°. 602,8 yards.

The logarithm verſed ſines on the callipers are the ſupplements of the real verſed ſines; therefore in the uſing of this line the ſupplements of the double angles are to be uſed.

Then the extent from the verſed ſine of 90° to the verſed ſine of 130° (the ſupplement of 50°) will on the line of numbers reach from $1687\frac{1}{2}$ to 603.

Or thus. Take $1292\frac{1}{2}=\frac{1}{4}$ of 5170, the horizontal range on an elevation of 25°.

Then. The extent on the log, tangents from radius to 25°, will on the line of numbers reach from $1292\frac{1}{2}$ to about 603 yards.

Examp. III. *At an elevation of 25 degrees, how many ſeconds will a 24 pounder continue in the air before it arrives at the horizon?*

At 45° elevation the ſhot takes $35\frac{1}{2}$ ſeconds in the air *.

And the times in air are as the right ſines of the elevations.

Then As the ſine of the elevation 45 degrees
Is to the ſine of the elevation 25 degrees
So is the time in air at 45°, viz. $35\frac{1}{2}$ ſeconds
To the time in air at 25°, viz. 21. ſeconds.

On the line of log. ſines take the extent from 45 degrees to 25 degrees; then will this extent, applied to the ſcale of log. numbers, reach from $35\frac{1}{2}$ to 21 ſeconds.

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* This time of $35\frac{1}{2}$ ſeconds is derived from Rule XV. Then working by the rules belonging to article XII. it will be found that a heavy body will require $35\frac{1}{2}$ ſeconds to fall through the ſpace of 6750 yards.

And hence may be eſtimated the lengths of fuſes proper for ſhells to be fired at given elevations and ranges.

Examp. IV. *Required the elevation neceſſary to ſtrike an object on the horizon at* 5170 *yards diſtance, the greateſt random of that piece being* 6750 *yards ?*

Say. As the greateſt random, 6750 yards
To a propoſed random, 5170 yards,
So is radius, or twice the ſine of 45 degrees,
To double the elevation required, viz. 50 deg.

The half of which, or 25 degrees, is the elevation neceſſary to be given to the piece.

This elevation is called the lower one.

And the upper elevation, is at 65 degrees.

For 25 degrees and 65 degrees are equally diſtant from 45 degrees.

Examp. V. *At an elevation of* 45 *degrees,* 16 ℔. *of powder will throw a* 24 *pounder* 6750 *yards: How much powder will throw the ſame ſhot* 5170 *yards at the ſame elevation ?*

By rule IX. The charges of powder are nearly as the horizontal ranges.

| | |
|---|---|
| Then As the horizontal range | 6750 |
| To the horizontal range | 5170, |
| So is the given charge | 16 ℔. |
| To the required charge | 12,26 ℔. |

This proportion may be accurately enough worked by the line of numbers.

For the extent from 6750 to 5170, will reach from 16 to 12$\frac{1}{4}$.

Examp. VI. *At an elevation of* 25 *degrees, a* 24 *pounder was ranged on the horizon* 5170 *yards : Required the impetus that would have given an equal velocity to that ſhot ?*

With an equal charge of powder uſed at 45 degrees of elevation, as was uſed at 25 degrees, the ſhot would have the greateſt horizontal range.

And with equal charges in the ſame piece, the impetus is the ſame at any elevation.

Consequently, to ſolve this queſtion nothing more is required than to find the greateſt horizontal range, which is double to the impetus.

Then from rule V, by inverſion
As the ſine of 50 deg. twice the given elevation,
Is to radius, or the ſine of twice 45°,
So is the given horizontal range 5170
To the greateſt horizontal range 6750,
The half, or 3375 is the impetus required.
That is, the extent on the line of ſines from 50° to 90°
Will on the line of numb. reach from 5170 to 6750.

Examp. VII. *Suppoſe the horizontal range of a piece to be 6750 yards: Required the angle of elevation proper to ſtrike an object 12° above the level of the piece, the horizontal diſtance of that object being 4680 yards?*

Say, As the greateſt horizontal range 6750
Is to the given horizontal diſtance 4680
So is the coſine of the object's elevation 78° 00′.
To another ſine —— — — —42° 42′.

Thus, the extent on the line of numbers from 6750 to 4680

Will on the line of log. ſines reach from 78° to about $42\frac{3}{4}$.

Now on the natural ſines, take the extent of $42\frac{3}{4}$ deg.

Then this extent applied from the natural ſine of the elevation 12°

Will give the natural ſine of about $62\frac{1}{2}$ degrees, whoſe coſine is about $27°\frac{1}{2}$.

Or rather 27°. 37′. its half is 13°. 48′.

The ſum of 90° and the given elevation 12° is 102; the half is 51°.

Then the ſum of theſe halves (51°+13°.48′=) 64° 48′, is the greater elevation.

And the difference of theſe halves (51°—13°. 48=) 37°. 12′. is the leſſer elevation.

So that the piece pointed at either of theſe elevations, with the charge of powder that gave the horizontal range, the object will be ſtruck.

But in all ſhooting on aſcents or deſcents, it is beſt to take the angle between the object and zenith, and get the complement of the half of that angle; then the piece being elevated to that complement, find by trials what charge will reach the object: For on this elevation, a leſs charge of powder will do the buſineſs than on any other elevation.

So in the foregoing example the diſtance of the object from the zenith is 78°,

The half of 78 is 39, and the complement of 39 is 51°, for the beſt elevation.

## ARTICLE XV.

### *Of the line of inches.*

This line, the uſe of which is well known, is placed on the edge of the callipers, or on the ſtrait borders of the faces c, d.

## ARTICLE XVI.

### *Of the logarithmic ſcales of numbers, ſines, verſed ſines and tangents.*

These ſcales are placed along the faces c, d of the callipers, near the ſtrait edges, and are marked and numbered as is ſhewn in ſection X; ſome of the uſes of theſe ſcales are alſo ſhewn in the XV and following ſections.

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## ARTICLE XVII.

### *Of the line of lines.*

THE line of lines is placed on the callipers on the faces C, D, in an angular poſition, tending towards the centre of the inſtrument; its conſtruction and uſes are the ſame as deſcribed in treating of the ſector; the reader will find ſufficient inſtructions in the ſections XI and XII.

## ARTICLE XVIII.

### *Of the lines of plans or ſuperficies.*

THESE lines lie on the faces C, D, of the callipers, and like the line of lines tend towards the centre of the inſtrument: They are marked near the ends of the callipers with the word *Plan*, and have the figures 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 1, running towards the centre: Each of theſe primary diviſions is ſubdivided into ten parts; and each of the ſubdiviſions is alſo divided into two, or more parts, according to the length of the callipers.

THESE diviſions reckoned from the centre along either leg, are as the ſquare roots of all the whole numbers under 100; and alſo, of the half numbers: That is, the diſtance from the centre to the firſt 1, is as the ſquare root of 1: From the centre to the next diviſion is as the ſquare root of 1½: To the next as 2, the next as 2½, the next as 3, &c.

AND the diſtance from the centre to the ſecond 1, is as the ſquare root of 10; from the centre to the next diviſion is as the ſquare root of 10½; to the next as 11; to the next as 11½, &c. So that the diſtances from the centre to 2, to 3, to 4, and ſo on to 10, are as the ſquare roots of 20, 30, 40, and ſo on to 100; and the intermediate diviſions and ſubdiviſions are eſtimated as before ſhewn between 1 and 10.

This line is eaſily conſtructed from a table of the ſquare roots of all the units and half units under 100; together with a ſcale of the intended length of the line of plans, divided into 500 or 1000 equal parts; and ſuch a ſcale is the line of lines.

In the followng ſolutions, the lengths of lines are ſuppoſed to be taken between the points of a pair of compaſſes: And when the callipers are ſaid to be opened to any line; it means, to the diſtance of the points of the compaſs between which that line was taken; the points being applied tranſverſely to the legs of the callipers, as ſhewn for the ſector at ſection XII.

### SOME USES OF THE SCALES OF PLANS.

Examp. I. *To find the ſquare root of a given number.*

1*ſt.* On the line of plans ſeek the diviſion repreſenting the given number: Obſerving, that numbers of an odd number of places are beſt found between the diviſions 1 and 1; and thoſe of an even number of places, between the 2d 1 and the 10 at the end.

2*d.* Take, with the compaſſes, the diſtance between that diviſion and the centre of the callipers; and this extent being applied, from the centre laterally along the *line of lines*, will give the ſquare root of the number propoſed.

Thus the ſquare root of 9 is 3
of 900 is 30
of 90000 is 300
&c. &c.

The given numbers being reckoned between the two diviſions marked 1 and 1.

Again the ſquare root of 36 is 6
of 360 is 18,9
of 3600 is 60
of 36000 is 189,7.

If the integer places in the given number are even, the root will conſiſt of half as many places: But if the number of integers be odd, increaſe it by one, and the integer places in the root will be half that number of places.

Thus numbers of two, four, ſix, eight integer places, will have roots conſiſting of one, two, three, four, &c. places: And numbers conſiſting of one, three, five, &c. places, have roots of one, two, three, &c. places.

Examp. II. *Between two given numbers (ſuppoſe 4 and 9) to find a mean proportional.*

1*ſt.* Take the greater of the given numbers (9) laterally from the line of lines, and make this extent a tranſverſe diſtance to (9 and 9) the ſame number on the lines of plans.

2*d.* Take the tranſverſe diſtance between (4 and 4) the leſſer given number on the lines of plans, and this extent applied laterally on the line of lines, will give (6 for) the mean proportional ſought.

For 4 : 6 :: 6 : 9.

By this example it is eaſy to ſee how to find the ſide of a ſquare equal to a ſuperficies whoſe length and breadth are given.

Examp. III. *Two ſimilar, or like, ſuperficies being given; to find what proportion they have to one another.*

1*ſt.* Take one ſide of the greater ſuperficies between the points of the compaſſes, and make this extent a tranſverſe diſtance on the line of plans between 10 and 10; or 100 and 100: or on any other number.

2*d.* Apply a like ſide of the leſs ſuperficies tranſverſely to the line of plans, and the diviſions it falls on will ſhew the number, that to the former number (taken tranſverſely for the ſide of the greater ſuperficies) bears the ſame proportion of the leſſer ſuperficies to the greater.

This propoſition may be wrought laterally on either of the legs, reckoning from the centre: For like ſides of ſimilar plans being laid from the centre on either leg, will give numbers ſhewing the proportion of thoſe plans.

EXAMP. IV. *To find the ſides, or other lines, of a ſuperficies* A, *which ſhall be ſimilar to a given ſuperficies* B, *and in a given proportion to* B, *ſuppoſe as* 3 *to* 7 *?*

1*ſt*. To the ſcales of plans, apply tranſverſely, any given line of B to the conſequent of the given ratio, as from 7 to 7.

2*d*. TAKE the tranſverſe diſtance, on the plans, of the given antecedent, as from 3 to 3, and this extent will be a like line of the figure A.

3*d*. As many lines being thus found as is neceſſary, the figure A may be conſtructed.

EXAMP. V. *To find the ſides, or other lines, of a ſuperficies* D, *which ſhall be like to either of two given plane figures* A *and* B; *and alſo be equal to the ſum or difference of* A *and* B.

1*ſt*. FIND (by Ex. 3.) two numbers expreſſing the proportion of the given figures A and B; and take the ſum and difference of thoſe numbers.

SUPPOSE the proportion of A to B, to be as 3 to 7.
THEIR ſum is 10, and their difference is 4.
THEN if D is to be like A.
FOR the ſum, it will be 3 : 10 : : A : D.
FOR the diff. it will be 3 : 4 : : A : D.
BUT if D is to be like B.
THEN, for the ſum, it will be 7 : 10 : : B : D.
AND for the diff. it will be 7 : 4 : : B : D.

2*d*. FIND (by Ex. 4.) the ſides of a ſuperficies D, ſimilar to A, ſo that A may be to D as 3 to 10 for the ſum, or as 3 to four for the difference; or if like to B, ſo that B may be to D as 7 to 10 for the ſum, or as 7 to 4 for the difference.

AND thus, a ſufficient number of lines being found the figure D may be conſtructed.

EXAMP. VI. *Three numbers being given to find a fourth in a duplicate proportion: Or, the like ſides* a, b, *of two ſimilar figures* A, B, *being known, and alſo the area* A, *of one, to find the area* B, *of the other.*

ON the ſcale of plans, take the given ſuperficies A laterally; and on the ſcale of lines, apply this diſtance tranſverſely to the given ſide *a* of that ſuperficies: Take the tranſverſe diſtance of the given ſide *b* of the other ſuperficies, from the ſcale of lines; then this diſtance applied laterally on the ſcale of plans, will ſhew the area of B.

THUS. *If* 40 *poles be the ſide of a ſquare whoſe area is* 10 *acres; what is the area of that ſquare whoſe ſide is* 60 *poles?*

TAKE the lateral diſtance 10 on the ſcale of plans; apply this diſtance tranſverſely to 40 and 40 on the line of lines: Then the tranſverſe diſtance of 60 and 60 on the lines, applied laterally to the ſcale of plans, will give 22½ acres the area required.

AGAIN. *How many acres of woodland meaſure, of* 18 *feet to the pole, is in that field which contains* 288 *acres, at* 16½ *feet to the pole?*

APPLY the lateral diſtance of 288, taken from the ſcale of plans, to the line of lines, tranſverſely from 18 to 18; then the tranſverſe diſtance of 16½ and 16½ on the lines, will, on the ſcale of plans, give 242 the area in woodland acres.

EXAMP. VII. *To open the callipers, ſo that the lines of plans make with one another a right angle?*

ON the line of plans take the lateral extent of any number thereon.

THEN ſet the callipers ſo, that this extent ſhall be a tranſverſe diſtance to the halves of the former number,

and

and the lines of plans will then ſtand at right angles to one another.

THUS: The lateral extent of 60 on the plans, put tranſverſely to 30 and 30 on the plans, will ſet thoſe lines at right angles to one another.

## ARTICLE XIX.

### *Of the line of ſolids.*

THESE lines are laid on the faces of D, C, of callipers, like ſectoral lines tending to the centre, and are diſtinguiſhed by the letters SOL placed at their ends.

THERE are twelve primary diviſions on theſe lines marked 1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; each of the eleven ſpaces or intervals is divided into ten other parts; and each of theſe parts is divided into two or more parts, according to the length of the inſtrument.

THESE diviſions are beſt taken from a ſcale of equal parts, ſuch as the line of lines, and thence transferred to the ſcales of ſolids, reckoning from the centre; from whence the ſeveral diſtances of the diviſions are, as the cube roots of ſuch numbers under 100 as are intended to be introduced.

THUS, the diſtance of the firſt 1 from the centre is as the cube root of $\frac{1}{10}$, and the greater diviſions following to the ſecond 1, expreſs the cube roots of $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, &c. to the number 1, which the ſecond 1 ſtands for; and if theſe ſpaces are ſubdivided, their diſtances from the centre are as the cube roots of $\frac{15}{100}$, $\frac{25}{100}$, $\frac{35}{100}$, $\frac{45}{100}$, &c.

THE diſtance from the centre to the ſecond 1 is as the cube root of 1, and the greater diviſions between the ſecond 1 and the third 1, are as the cube roots of the whole numbers 2, 3, 4, 5, 6, 7, 8, 9; the intermediate ſmaller diviſions are as the cube roots of the mixed numbers to which they belong: Thus if the ſpace between the diviſions repreſenting the roots of 1 and 2 is parted into 4; then thoſe ſubdiviſions will be as the cube roots of $1\frac{25}{100}$, $1\frac{5}{10}$, $1\frac{75}{100}$; and the like for other ſubdiviſions.

THE diſtance between the centre and the third 1 is as the cube root of 10; and ſo the following diviſions marked with 2, 3, 4, &c, to 10, are as the cube roots of 20, 30, 40, &c. to 100: each of theſe ſpaces are divided into 10 parts, which are as the cube roots of the intermediate whole numbers; and if theſe ſubdiviſions are again divided, theſe latter diviſions will be as the cube roots of the mixed numbers to which they belong.

ON the French inſtruments, the diviſions of this line is uſually extended to 64; and conſequently only the cube roots of all the integer numbers under 64 are thereon expreſſed: Now whether the diviſions proceed only to 64 or to 100, the beſt way of laying them down is from a table of cube roots ready computed, reckoning the length of the greateſt root, or the length of the ſcale of ſolids, to be equal to the length of the line of lines, taken from the centre.

THE cube roots of $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, are, 0,464. 0,585. 0,669. 0,737. 0,794. 0,843. 0,888. 0,928. 0,965.

THE following table contains the cube roots of all the whole numbers from 1 to 100.

## TABLE *of cube numbers and their roots.*

| Cubes | Roots | Cubes | Roots | Cubes | Roots | Cubes | Roots | Cubes | Roots |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1,000 | 21 | 2,759 | 41 | 3,448 | 61 | 3,936 | 81 | 4,327 |
| 2 | 1,260 | 22 | 2,802 | 42 | 3,476 | 62 | 3,958 | 82 | 4,344 |
| 3 | 1,442 | 23 | 2,844 | 43 | 3,503 | 63 | 3,979 | 83 | 4,362 |
| 4 | 1,587 | 24 | 2,884 | 44 | 3,530 | 64 | 4,000 | 84 | 4,379 |
| 5 | 1,710 | 25 | 2,924 | 45 | 3,557 | 65 | 4,021 | 85 | 4,397 |
| 6 | 1,817 | 26 | 2,962 | 46 | 3,583 | 66 | 4,041 | 86 | 4,414 |
| 7 | 1,913 | 27 | 3,000 | 47 | 3,609 | 67 | 4,061 | 87 | 4,431 |
| 8 | 2,000 | 28 | 3,036 | 48 | 3,634 | 68 | 4,082 | 88 | 4,448 |
| 9 | 2,080 | 29 | 3,072 | 49 | 3,659 | 69 | 4,102 | 89 | 4,465 |
| 10 | 2,154 | 30 | 3,107 | 50 | 3,684 | 70 | 4,121 | 90 | 4,481 |
| 11 | 2,224 | 31 | 3,141 | 51 | 3,708 | 71 | 4,141 | 91 | 4,498 |
| 12 | 2,285 | 32 | 3,175 | 52 | 3,732 | 72 | 4,160 | 92 | 4,514 |
| 13 | 2,351 | 33 | 3,207 | 53 | 3,756 | 73 | 4,179 | 93 | 4,531 |
| 14 | 2,410 | 34 | 3,240 | 54 | 3,780 | 74 | 4,198 | 94 | 4,547 |
| 15 | 2,466 | 35 | 3,271 | 55 | 3,803 | 75 | 4,217 | 95 | 4,563 |
| 16 | 2,520 | 36 | 3,302 | 56 | 3,826 | 76 | 4,236 | 96 | 4,579 |
| 17 | 2,571 | 37 | 3,332 | 57 | 3,848 | 77 | 4,254 | 97 | 4,595 |
| 18 | 2,621 | 38 | 3,362 | 58 | 3,871 | 78 | 4,273 | 98 | 4,610 |
| 19 | 2,668 | 39 | 3,391 | 59 | 3,893 | 79 | 4,291 | 99 | 4,626 |
| 20 | 2,714 | 40 | 3,420 | 60 | 3,915 | 80 | 4,309 | 100 | 4,642 |

THE numbers in the foregoing table may be laid on the line of ſolids in the following manner.

MAKE the length of the line of ſolids equal to the length of the line of lines, apply this extent tranſverſely to 4,642 on the line of lines; then the other numbers in the table taken tranſverſely from the line of lines, are to be laid laterally, from the centre, on the line of ſolids.

### *Some uſes of the lines of ſolids.*

EXAMP. I. *To find the cube root of a given number.*

SEEK the given number on the line of ſolids, and take its extent from the centre.

THEN this extent applied laterally to the line of lines will give the cube root ſought.

It ſhould be remarked, that a given number of
of 1, 2 or 3 places, has a root of one place.
of 4, 5 or 6 places, has a root of two places.
of 7, 8 or 9 places, has a root of three places.

And when a given number is ſought for on the line of ſolids,

The primary diviſions from 1 to 10 may be reckoned either as ſo many hundreds, or as ſo many hundred thouſands, or as ſo many hundred millions.

Thus the diviſion marked 5 may either repreſent 500, or 500000, or 500000000.

And the like of the other primary diviſions and their intermediates.

And hence the diviſions between the centre and the firſt of the primary ones, are to be eſtimated for numbers of one, two, four, five, ſeven and eight places.

Examp. II. *To a number given, to find another in a triplicate ratio of two given numbers.*

Thus. Suppoſe a ſhot of 4 inches diameter to weigh 9℔; required the weight of that ſhot which is 8 inches in diameter?

Here a number is to be found, that to 9 ſhall be in the triplicate ratio of 4 to 8.

That is, as the cube of 4 is to the cube of 8, ſo is 9 to the number ſought.

Now from any ſcale of equal parts, ſuppoſe inches, take 4; and make it a tranſverſe diſtance to 9 and 9 on the line of ſolids (reckoning the 10 at the end, as 100): Then will the extent of 8 inches, applied tranſverſely to the line of ſolids, give 72 for the number ſought, which is the pounds weight of a ſhot of 8 inches diameter.

AGAIN. *Suppoſe a ſhip of* 2000 *tons burthen is* 144 *feet* 6 *inches on the keel, and* 51 *feet by the beam: Required the length and breadth of another ſimilar ſhip that ſhall be of* 1415 *tons burthen?*

FROM any ſcale of equal parts take 144½ and make this extent a tranſverſe diſtance to 2000 on the line of ſolids; then will the tranſverſe diſtance of 1415 taken on the line of ſolids give the length of the keel, which applied to the ſaid ſcale of equal parts will give about 128¾ feet.

ALSO the extent in equal parts of 51 being made a tranſverſe diſtance to 2000 on the lines of ſolids; then the tranſverſe diſtance on the ſolids of 1415 will give in equal parts 46⅓ feet for the breadth by the beam.

EXAMP. III. *Between two given numbers or lines to find two mean proportionals.*

1*ſt*. FROM any ſcale of equal parts take the meaſure of the greateſt of the given lines or numbers, and apply this extent tranſverſely to that number on the line of ſolids; then the tranſverſe extent on the ſolids, of the leaſt of the given numbers, being taken, will be the greater of the required means, whoſe meaſure will be found on the ſaid ſcale of equal parts.

2*d*. MAKE the extent of the greater mean, a tranſverſe diſtance to the greater of the given numbers, on the line of ſolids; then the tranſverſe diſtance of the leſſer of the given numbers, taken from the line of ſolids, will give the leſſer of the required means.

*Suppoſe two mean proportionals were required between* 9 *and* 41⅔.

THE lateral extent of 41⅔, taken from the line of lines, apply tranſverſely to 41⅔ and 41⅔ on the line of ſolids; then the tranſverſe extent of 9 and 9 taken on the ſolids, and applied laterally to the line of lines will give 25 for the greater of the two means.

Apply the ſaid extent of 25 tranſverſely to $41\frac{2}{3}$ and $41\frac{2}{3}$ on the line of ſolids; then the tranſverſe extent on the ſolids from 9 to 9 applied laterally to the line of lines, will give 15 for the leſſer mean.

For 9, 15, 25 and $41\frac{2}{3}$ are in continual proportion.

Examp. IV. *To find the ſide of a cube equal to a parallelopipedon whoſe length, breadth and depth are given.*

*1ſt.* Between the breadth and depth find a mean proportional by Ex. 2. Art. 18.

*2d.* Find the meaſure of the mean proportional on the line of lines, and apply it to the lines of ſolids tranſverſely, at the numbers expreſſing that meaſure: Then the tranſverſe extent of the length being taken from the line of ſolids and applied laterally to the line of lines, will give the ſide of a cube equal to that parallelopipedon.

Thus, *Suppoſe a parallelopipedon, whoſe length is* 72, *breadth* 64, *and depth* 24.

The number 64 taken laterally from the line of lines and applied tranſverſely to 64 and 64 on the line of plans; then the tranſverſe diſtance of 24 and 24 on the plans meaſured laterally on the line of lines gives about 39,2 for the mean proportional.

Apply the extent of the mean proportional, to 39,2 tranſverſely on the line of ſolids; then the tranſverſe extent of 72 and 72 on the ſolids, being applied to the line of lines laterally, will give 48 for the ſide of the cube equal in ſolidity to the given parallelopipedon.

For $48\times48\times48=24\times64\times72=110592$.

Examp. V. *Two ſimilar ſolids* A *and* B *being given, to find their ratio.*

*1ſt.* Take any ſide of the ſolid A, and apply it tranſverſely on the line of ſolids from 10 to 10, or from any other number to its oppoſite.

2*d*. Apply the like ſide of the ſolid B tranſverſely to the lines of ſolids, and obſerve the number it falls on: Then will the numbers on which thoſe tranſverſe extents fall, ſhew the ratio of the ſolids A and B.

Examp. VI. *A ſolid* A *being given to find the dimenſions of a ſimilar ſolid* B, *that to* A *ſhall have any aſſigned ratio.*

1*ſt*. On the line of ſolids ſeek two numbers expreſſing the terms of the given ratio.

2*d*. Take the extent of one ſide of the given ſolid A, and apply it tranſverſely on the lines of ſolids to the antecedent of that given ratio; then the tranſverſe extent of the conſequent taken on the lines of ſolids will be a like ſide of the ſolid B.

Thus. *To find the ſide of a cube* B, *double to a given cube* A.

Here the ratio is as 1 to 2.

Apply the ſide of the cube A to the lines of ſolids tranſverſely from 1 to 1; that is from 10 to 10; then will the tranſverſe diſtance of the numbers 2 and 2 or 20 and 20 ſhew the ſide of the cube B.

Again. *To find the diameter of a ſphere* B, *that to the ſphere* A, *whoſe diameter is given, ſhall be in the ratio of* 3 *to* 2.

Make the diameter of the ſphere A a tranſverſe diſtance to 2 and 2 on the lines of ſolids; then will the tranſverſe diſtance of 3 and 3 on the line of ſolids be the diameter of the ſphere B.

Examp. VII. *Any number of unequal ſimilar ſolids being given; to find the ſide of a ſimilar ſolid equal in magnitude to the ſum of the magnitudes of the given ſolids.*

Take, in equal parts, a number expreſſing the ſide of one of the given ſolids, and apply this extent to the line of ſolids tranſverſely, to any number (ſuppoſe 10 at the 3d 1).

Also take in the ſame equal parts, the numbers ſhewing the ſimilar ſides of the other ſolids, and apply theſe extents to the lines of ſolids tranſverſely, noting the numbers they fall on.

Then will the tranſverſe extent on the line of ſolids of a number equal to the ſum of the noted numbers, be the like ſide of the ſimilar ſolid required, which applied to the ſame ſcale of equal parts the others were taken from will give the meaſure of that ſide.

Thus. *What will be the diameter of that iron ſhot caſt from 3 other ſhot whoſe diameters were 4 inches, 4, 4 inches, and 5 inches; ſuppoſing no waſte in melting?*

Make 4 inches a tranſverſe extent on the line of ſolids, to any number ſuppoſe 10. Then 4,4 inches applied tranſverſely to the ſolids will give about $13\frac{1}{3}$; and 5 inches alſo applied tranſverſely to the ſolids will give about $19\frac{2}{3}$: Now the ſum of the noted numbers 10 and $13\frac{1}{3}$ and $19\frac{2}{3}$ will be 43; then the tranſverſe extent of 43 on the line of ſolids will give $6\frac{1}{2}$ inches for the diameter of the new ſhot.

Examp. VIII. *To find the dimenſions of a ſolid which ſhall be equal to the difference of two given ſimilar ſolids, and alſo ſimilar to them.*

Apply a dimenſion of one ſolid tranſverſely to the line of ſolids at any number; and alſo note what number on the line of ſolids, the like dimenſion of the other ſolid falls tranſverſely on; take the difference of thoſe noted numbers; and on the line of ſolids take tranſverſely the extent of the remainder, and that will be a like dimenſion of the ſimilar ſolid required.

THUS. *With the powder out of a ſhell of* 10 *inches concave diameter is filled a ſhell of* 7 *inches* : *What ſized ſhell will the remaining powder fill ?*

THE extent of 10 inches being applied tranſverſely to the lines of ſolids, at any number ſuppoſe 100; the extent of 7 inches will fall tranſverſely on the lines of ſolids, about the number 34$\frac{1}{4}$: The difference between 100 and 34$\frac{1}{4}$ is 65$\frac{3}{4}$: Then the tranſverſe extent at 65$\frac{3}{4}$ on the line of ſolids, will give 8,7 inches for the concavity of that ſhell which the remaining powder will fill.

*FINIS.*

* * *

Pl. IV. *At the end.*

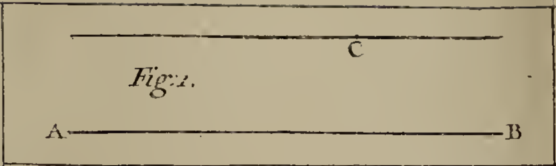


*Fig: 1.*

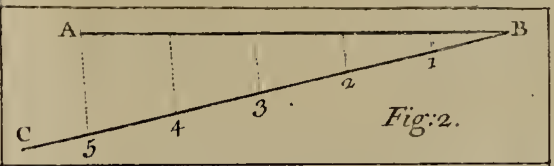


*Fig: 2.*

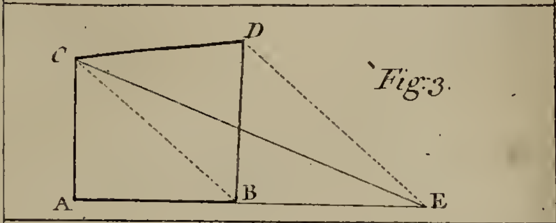


*Fig: 3.*

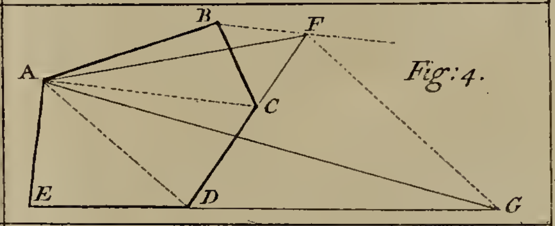


*Fig: 4.*

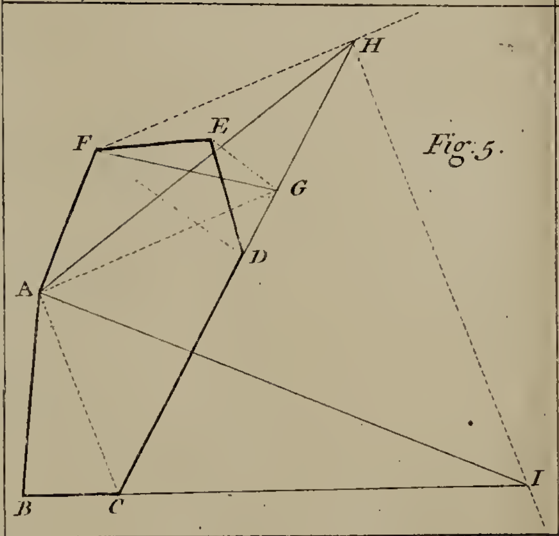


*Fig: 5.*

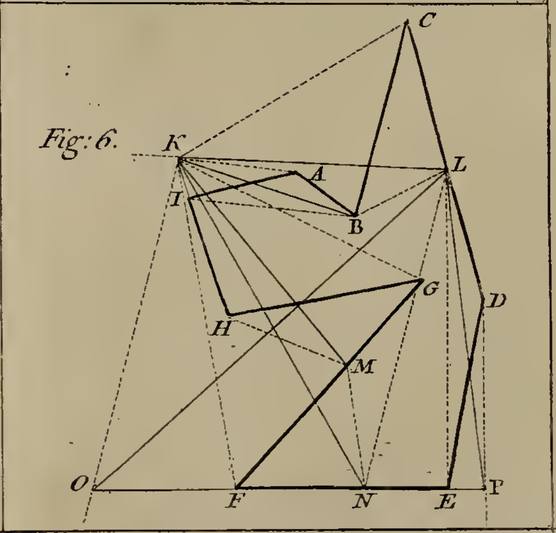


*Fig: 6.*

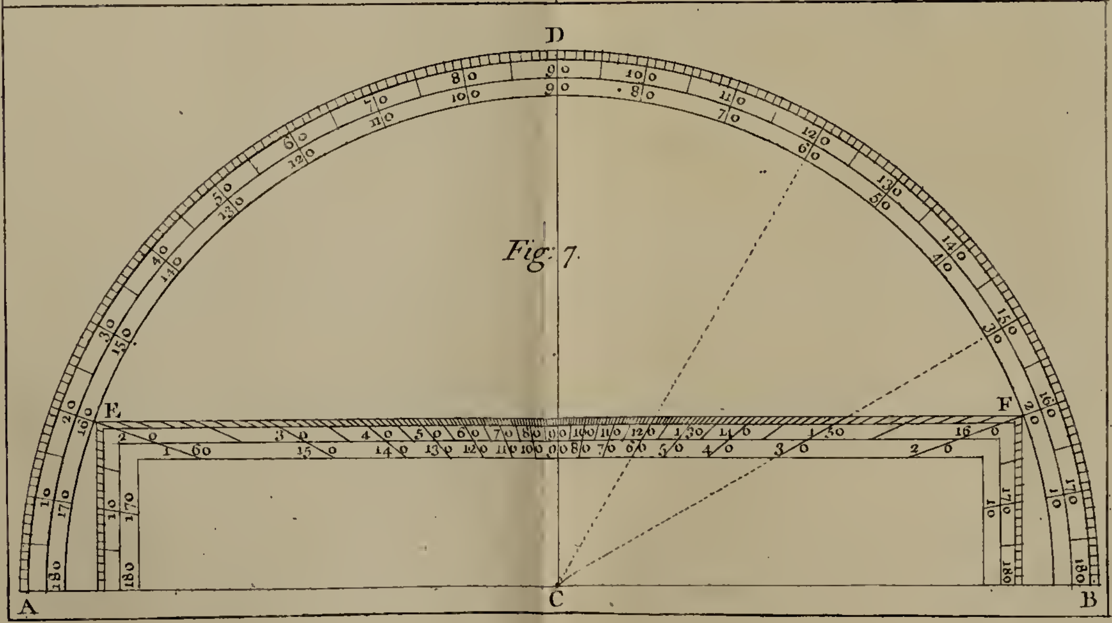


*Fig: 7.*

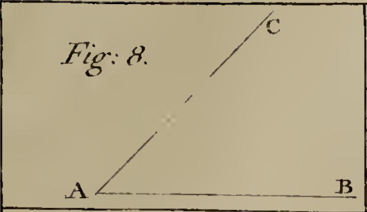


Fig: 8.

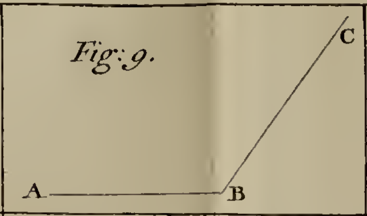


Fig: 9.

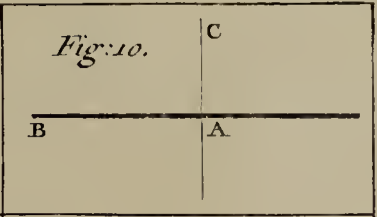
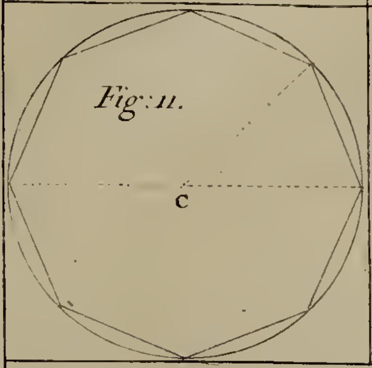


Fig: 10.



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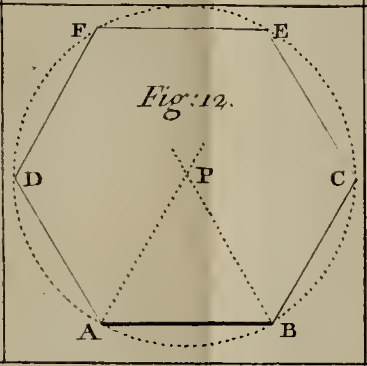

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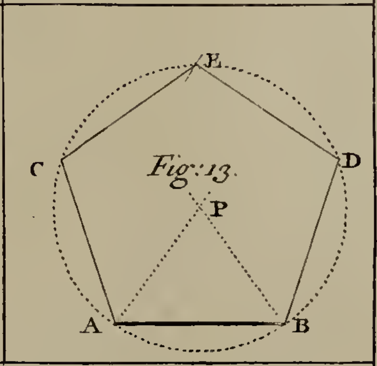

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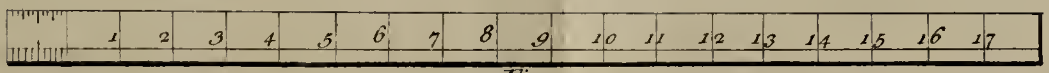
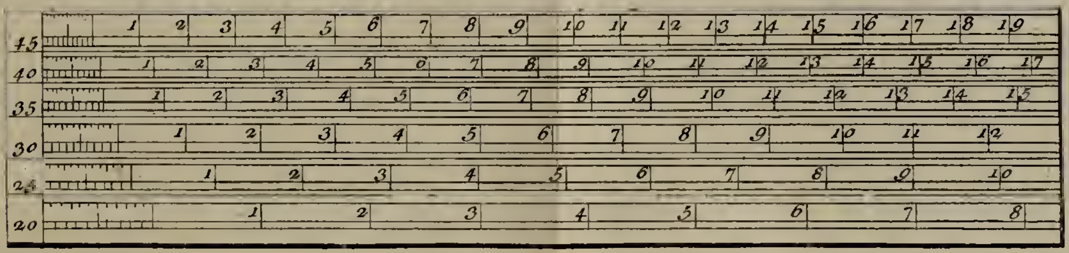


Fig: 14.



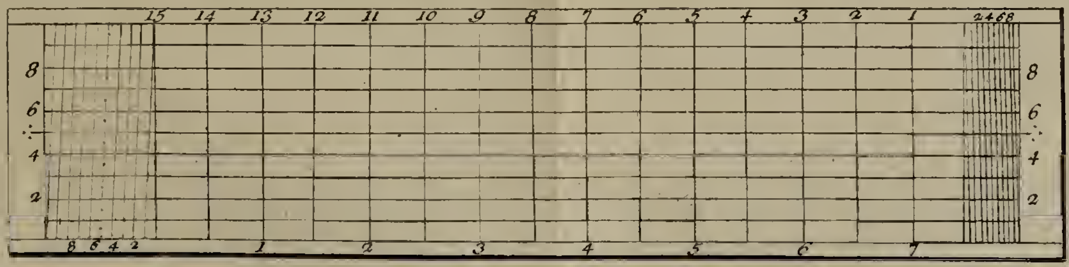


Fig: 15.

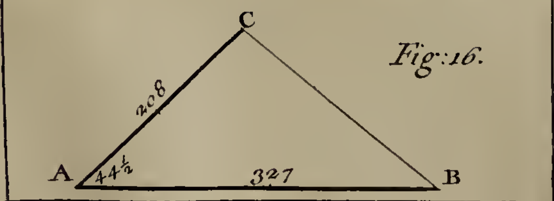


Fig: 16.

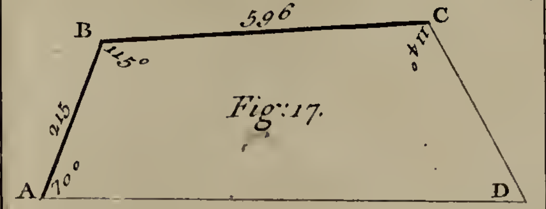
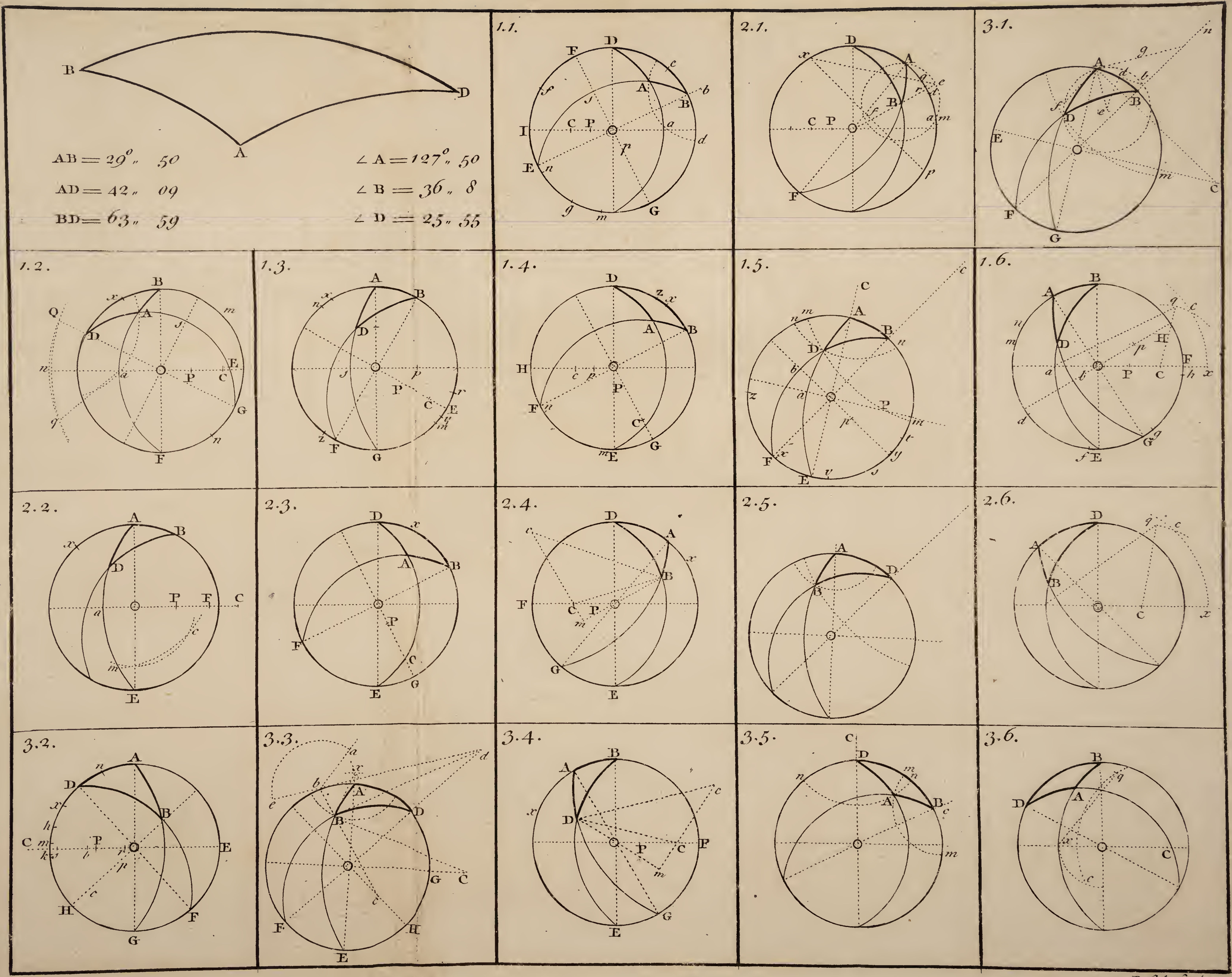


Fig: 17.

Seale Sculp.



AB = 29°. 50 ∠ A = 127°. 50

AD = 42. 09 ∠ B = 36. 8

BD = 63. 59 ∠ D = 25. 55

B. Cole sculp.

Fig. 18

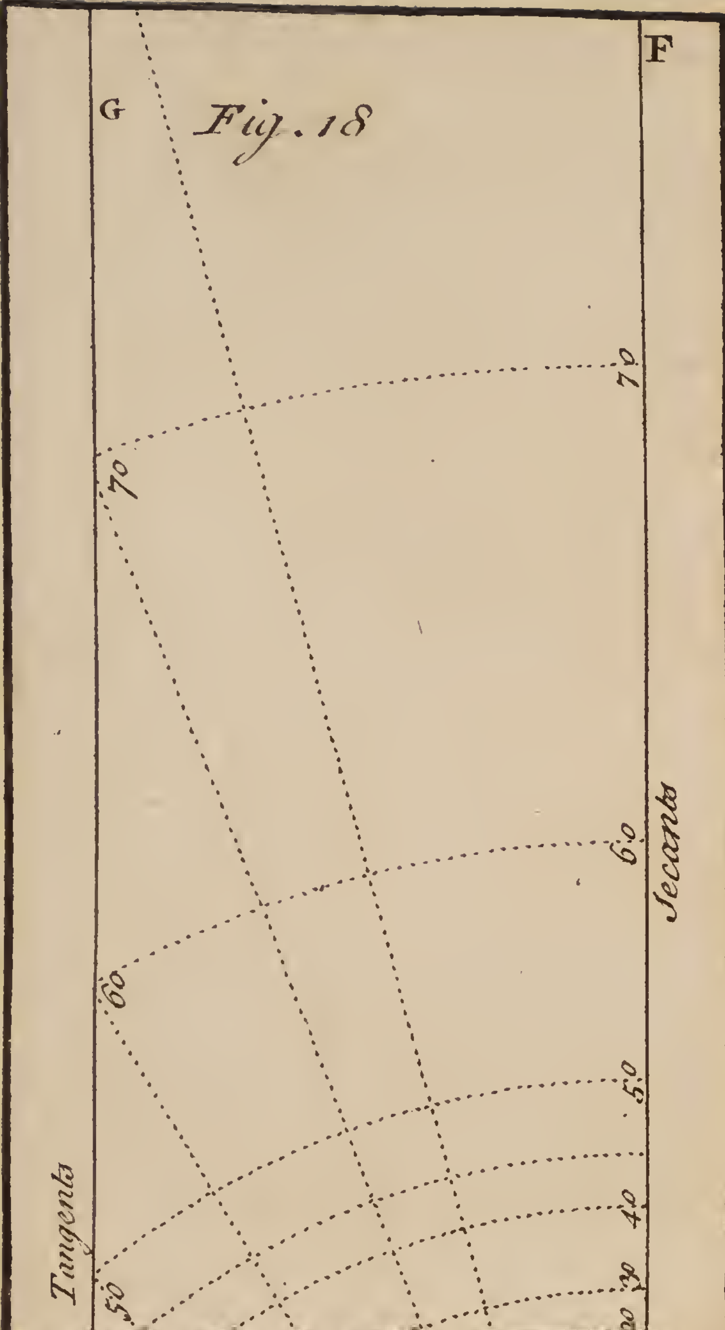
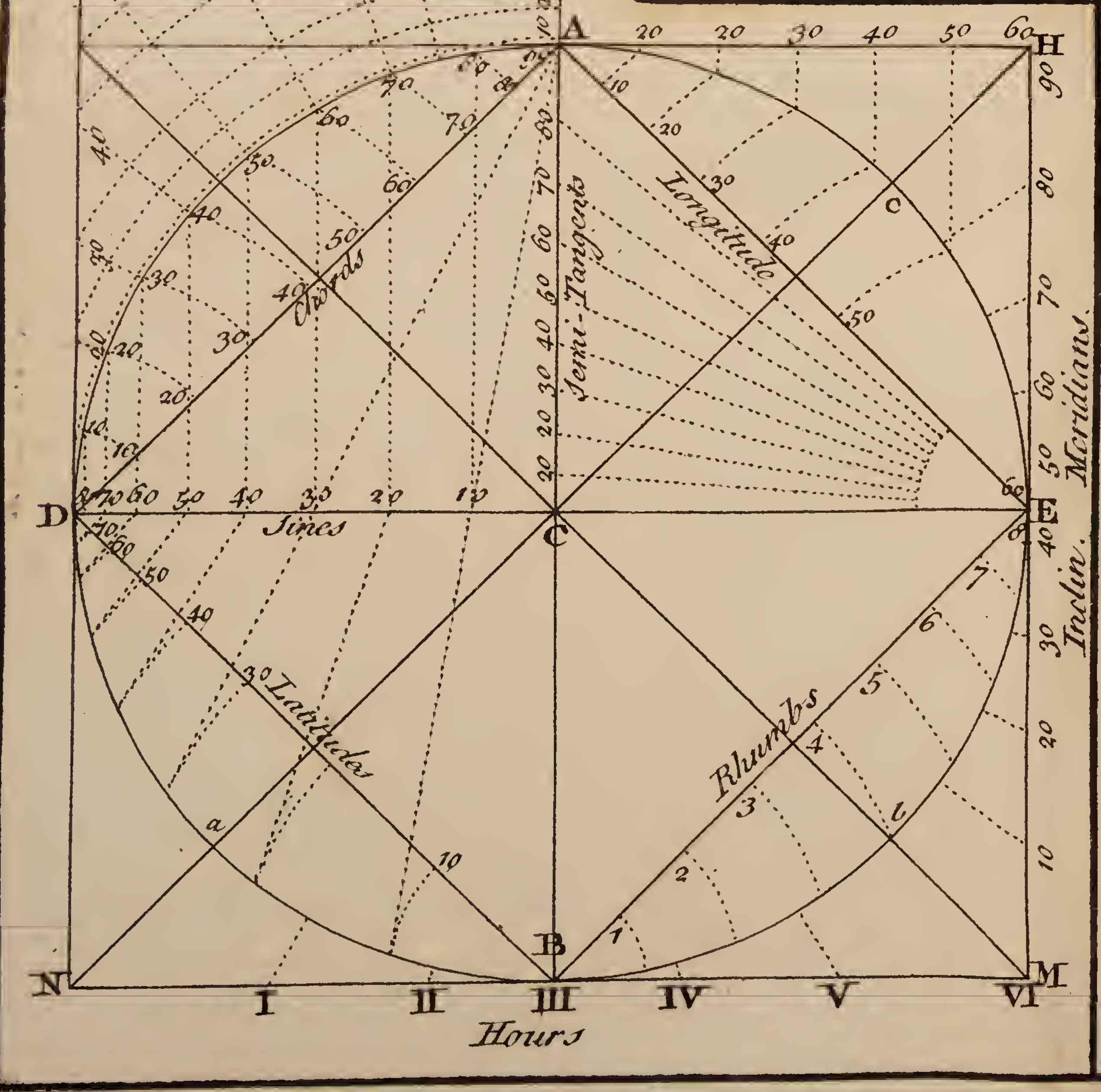



Fig. 19

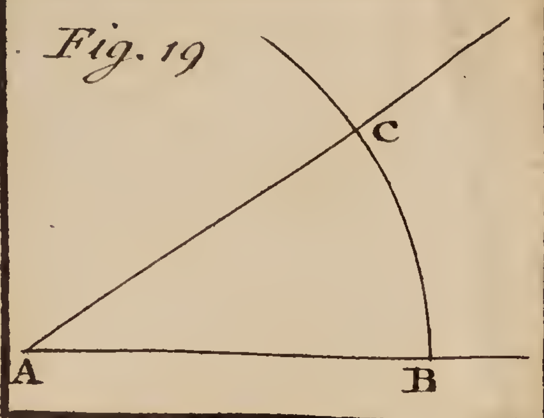


Fig. 20

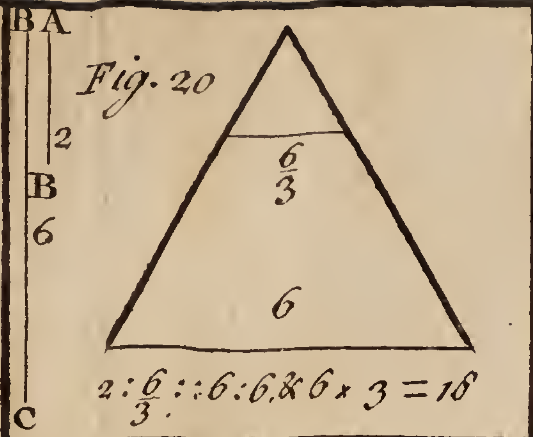


Fig. 21

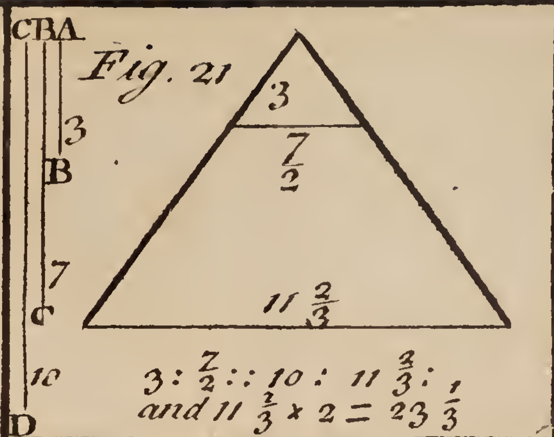


Fig. 22

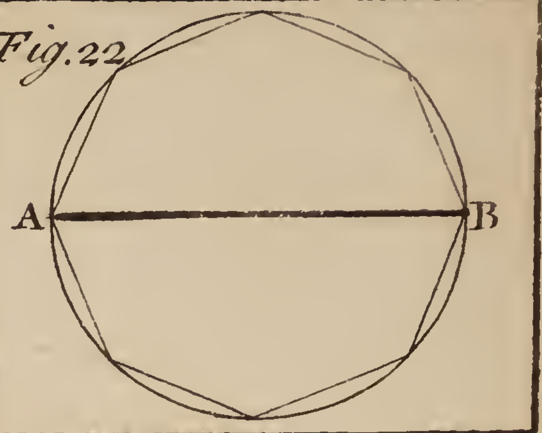


Fig. 23

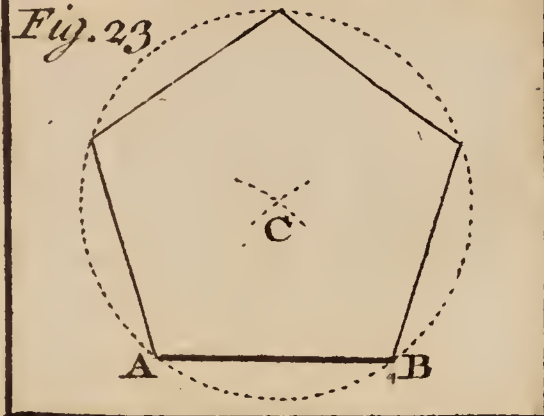


Fig. 24

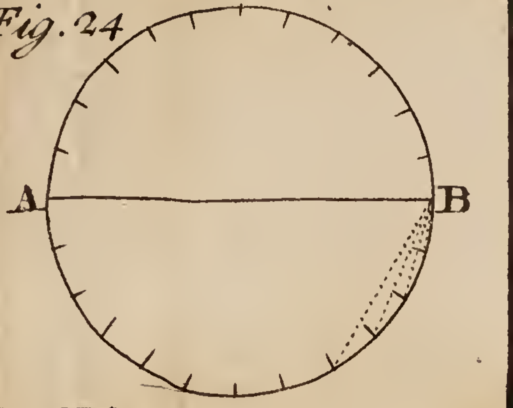


Fig. 25

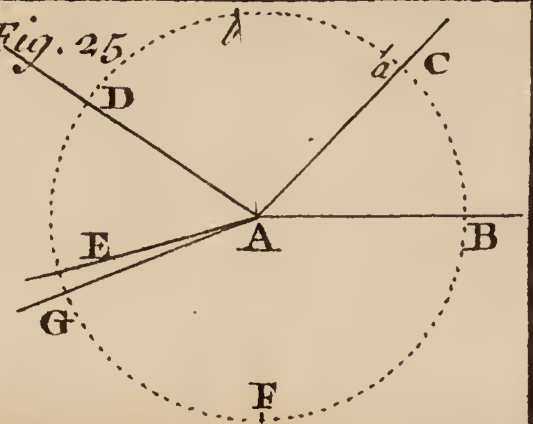


Fig. 26

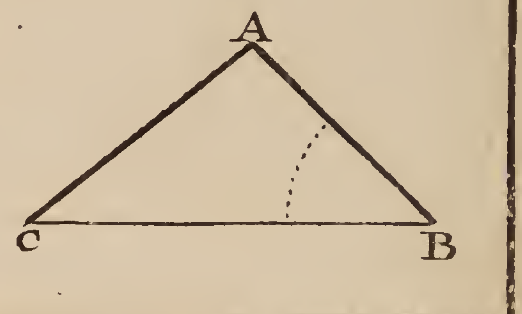


Fig. 27

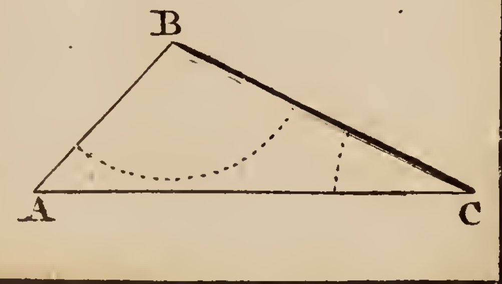


Fig. 28

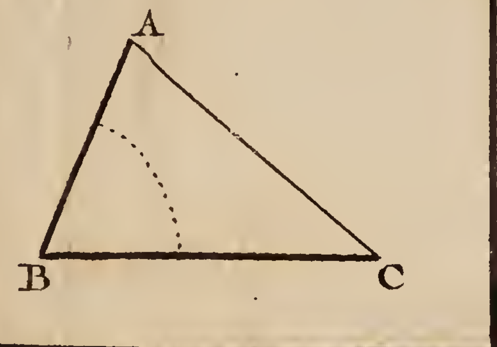


Fig. 29

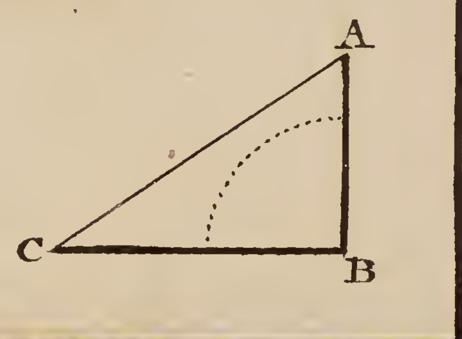


Fig. 30

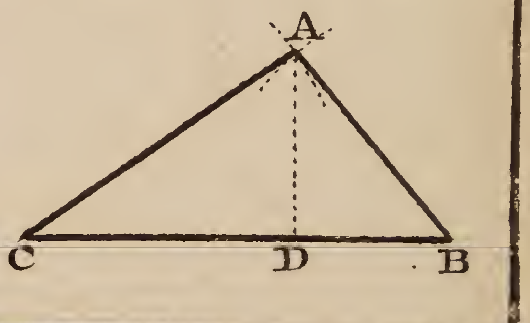
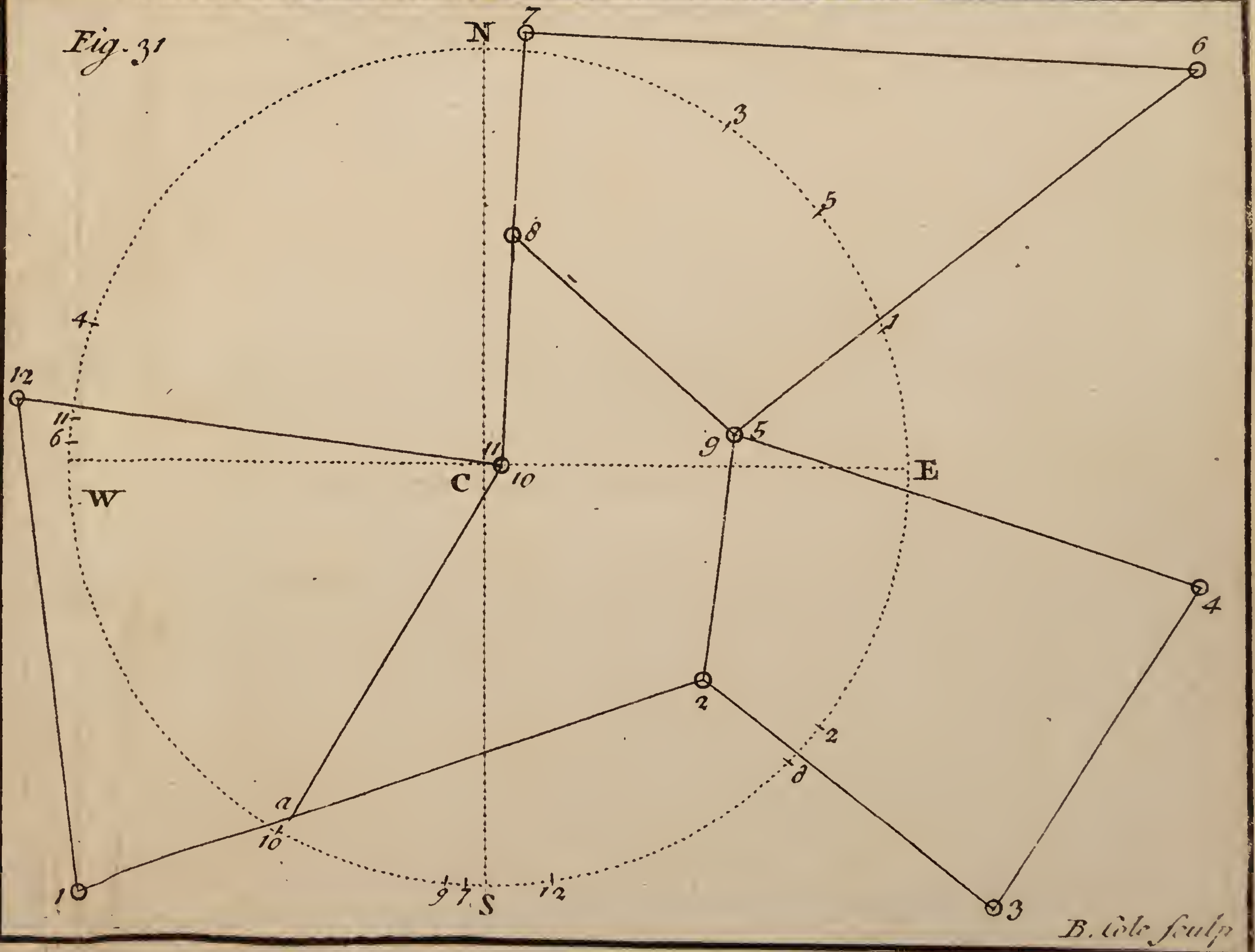
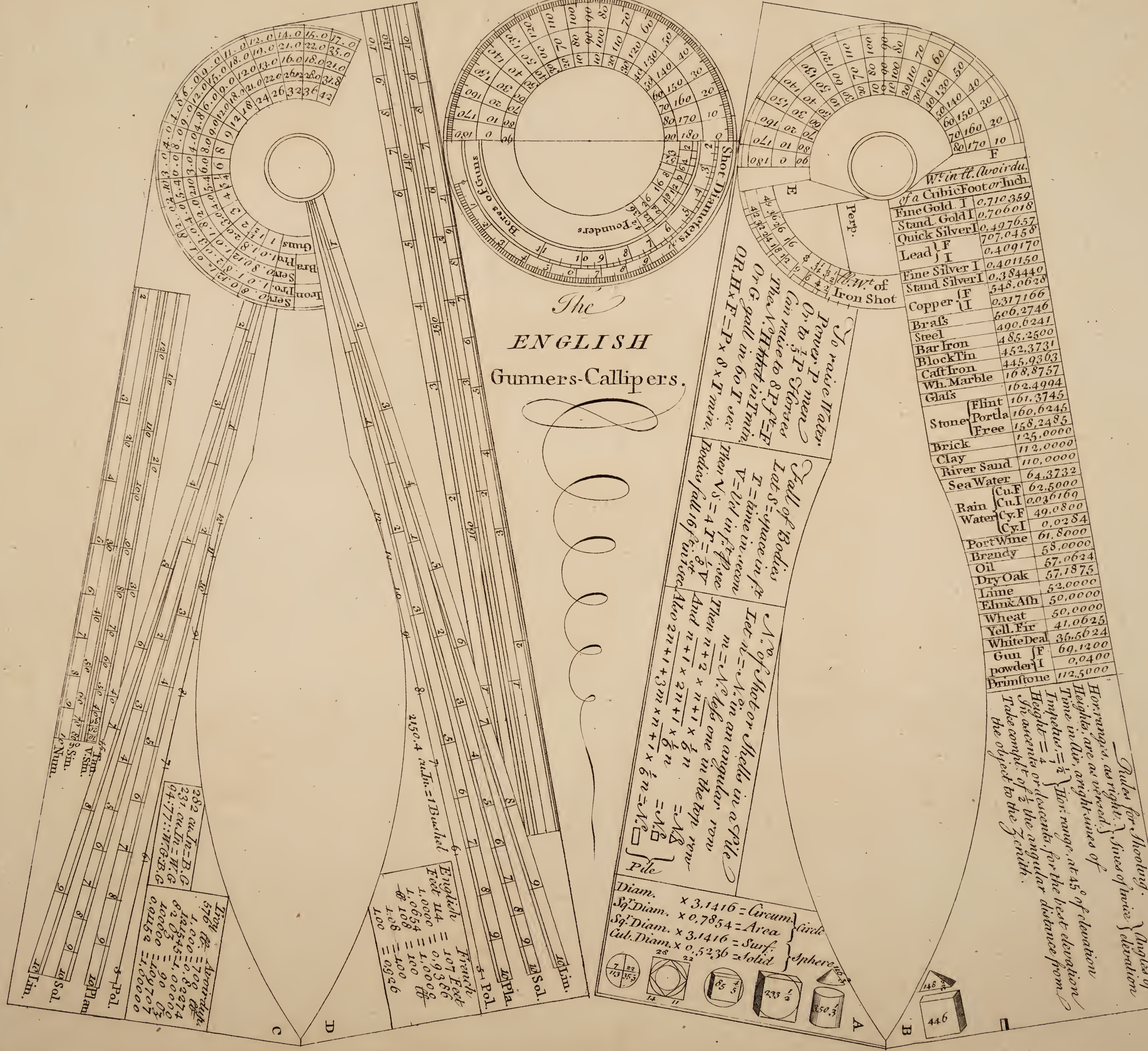


Fig. 31



B. Cole sculp

Pl.VIII at end



# The ENGLISH Gunners-Callipers.

| Wt. in lb. Avoirdu. of a Cubic Foot or Inch | | |
|---|---|---|
| Fine Gold. | I | 0,710359 |
| Stand. Gold | I | 0,706018 |
| Quick Silver | I | 0,497657 |
| Lead | F | 707,0458 |
| | I | 0,409170 |
| Fine Silver | I | 0,401150 |
| Stand Silver | I | 0,384440 |
| Copper | F | 548,0628 |
| | I | 0,317166 |
| Braſs | | 606,2746 |
| Steel | | 490,6241 |
| Bar Iron | | 485,2500 |
| Block Tin | | 452,3731 |
| Caſt Iron | | 445,9363 |
| Wh. Marble | | 168,8757 |
| Glaſs | | 162,4994 |
| Stone | Flint | 161,3745 |
| | Portla | 160,6245 |
| | Free | 158,2485 |
| Brick | | 125,0000 |
| Clay | | 112,0000 |
| River Sand | | 110,0000 |
| Sea Water | | 64,3732 |
| Rain Water | Cu.F | 62,5000 |
| | Cu.I | 0,036169 |
| | Cy.F | 49,0800 |
| | Cy.I | 0,0284 |
| Port Wine | | 61,8000 |
| Brandy | | 58,0000 |
| Oil | | 57,0624 |
| Dry Oak | | 57,1875 |
| Lime | | 52,0000 |
| Elm & Aſh | | 50,0000 |
| Wheat | | 50,0000 |
| Yell. Fir | | 41,0625 |
| White Deal | | 35,5624 |
| Gun powder | F | 69,1200 |
| | I | 0,0400 |
| Brimſtone | | 112,5000 |

To raise Water.
Power P men
Or to $\frac{1}{5}$ P Horses
Can raise to 8 P feet = F
The No. Hhds in T min.
Or G. gall in 60 T sec.
OR. H × F = P × 8 × T min.

Fall of Bodies
Let S = space in feet
T = time in secon
V = Vel in feet per sec
Then $\sqrt{S} = 4\,T = \frac{1}{8}\,V$
Bodies fall 16 feet in 1st sec.

No. of Shot or Shells in a Pile
Let n = No. in an angular row
m = No. less one in the top row
Then $\overline{n+2} \times \overline{n+1} \times \frac{1}{6}\,n$ = No. △
And $\overline{n+1} \times \overline{2n+1} \times \frac{1}{6}\,n$ = No. □
Also $\overline{2n+1+3m} \times \overline{n+1} \times \frac{1}{6}\,n$ = No. ▭ } Pile

Diam. × 3,1416 = Circum. } Circle
Sqr. Diam. × 0,7854 = Area }
Sqr. Diam. × 3,1416 = Surf. } Sphere
Cub. Diam. × 0,5236 = Solid }

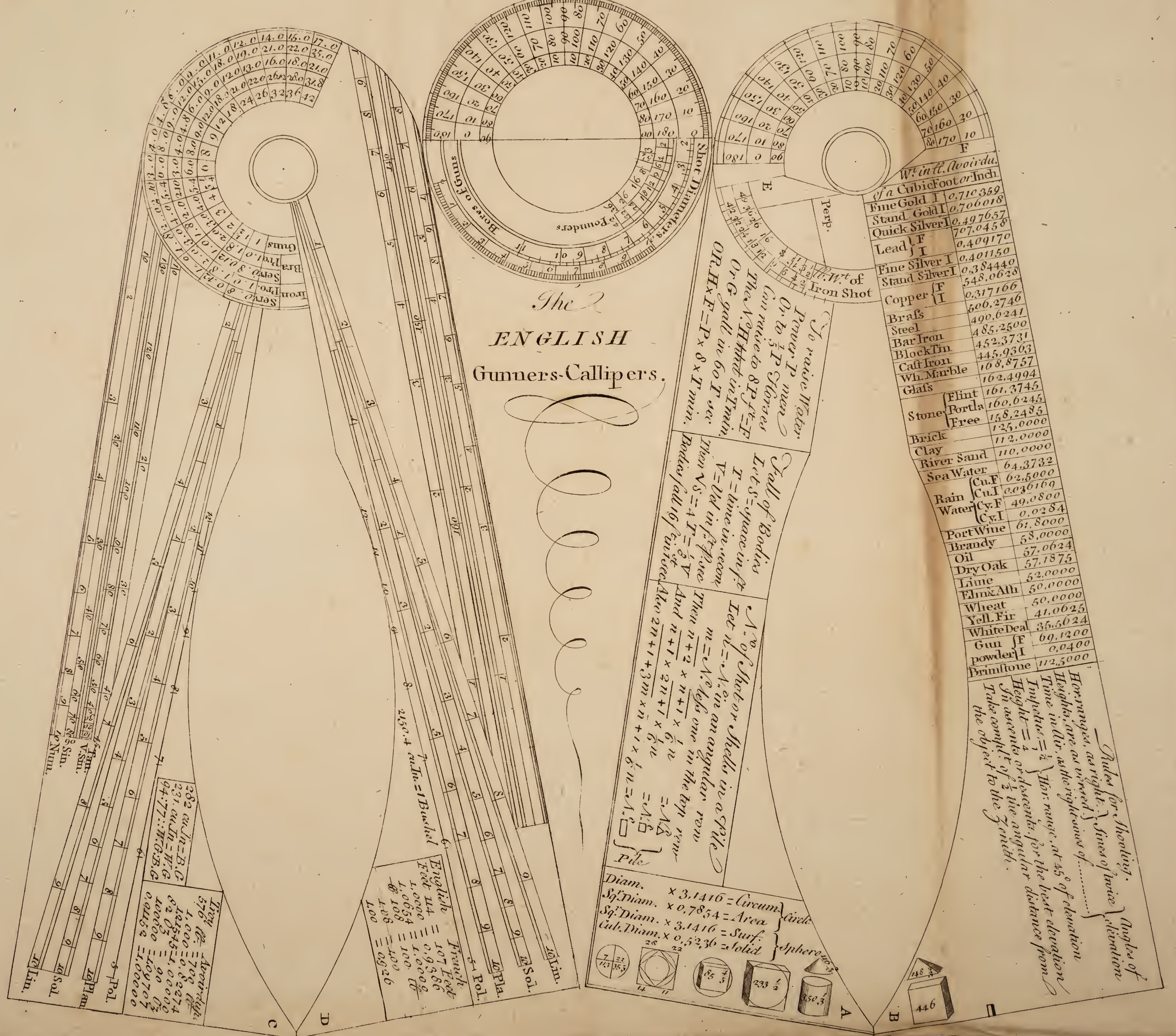
Rules for Shooting.
Hori: ranges, as right } Sines of twice } Angles of
Heights, are as versed } elevation
Time in Air, as right sines of
Impetus = 1/2 } Hori: range, at 45° of elevation
Height = 1/4 }
In ascents or descents, for the best elevation
Take complt. of 1/2 the angular distance from the object to the Zenith.

282 cu. In. = B.G
231 cu. In. = W.G
94 : 77 :: W.G : B.G

| Troy | Avoirdup. |
|---|---|
| 576 lb. | = 700 lb. |
| 1,000 | = 0,82274 |
| 1,21545 | = 1,00000 |
| 82 oz | = 90 oz |
| 1,00000 | = 1,09707 |
| 0,91152 | = 1,00000 |

2150,4 cu. In. = 1 Bushel

| English | French |
|---|---|
| Feet 114 | = 107 Feet |
| 1,0000 | = 0,93866 |
| 1,0654 | = 1,00000 |
| lb. 108 | = 100 lb. |
| 1,08 | = 1,00 |
| 1,00 | = 0,926 |



# The ENGLISH Gunners-Callipers.

| Wt. in lb. Avoirdu. of a Cubic Foot or Inch | |
|---|---|
| Fine Gold I | 0,710359 |
| Stand. Gold I | 0,706018 |
| Quick Silver I | 0,497657 |
| Lead F | 707,0458 |
| Lead I | 0,409170 |
| Fine Silver I | 0,401150 |
| Stand. Silver I | 0,384440 |
| Copper F | 548,0628 |
| Copper I | 0,317166 |
| Brass | 606,2746 |
| Steel | 490,6241 |
| Bar Iron | 485,2500 |
| Block Tin | 452,3731 |
| Cast Iron | 445,9363 |
| Wh. Marble | 168,8757 |
| Glass | 162,4994 |
| Stone Flint | 161,3745 |
| Stone Portla | 160,6245 |
| Stone Free | 158,2485 |
| Brick | 125,0000 |
| Clay | 112,0000 |
| River Sand | 110,0000 |
| Sea Water | 64,3732 |
| Rain Water Cu.F | 62,5000 |
| Rain Water Cu.I | 0,036169 |
| Rain Water Cy.F | 49,0800 |
| Rain Water Cy.I | 0,0284 |
| Port Wine | 61,8000 |
| Brandy | 58,0000 |
| Oil | 57,0624 |
| Dry Oak | 57,1875 |
| Lime | 52,0000 |
| Elm & Ash | 50,0000 |
| Wheat | 50,0000 |
| Yell. Fir | 41,0625 |
| White Deal | 35,5624 |
| Gun powder F | 69,1200 |
| Gun powder I | 0,0400 |
| Brimstone | 112,5000 |

Wt. of Iron Shot

Rules for Shooting.

Hor. ranges, as right. } Sines of twice } Angles of elevation
Heights, are as versed. }
Time in Air, as the right sines of ……… }
Impetus = $\frac{1}{2}$ } Hor. range, at 45° of elevation
Height = $\frac{1}{4}$ }
In ascents or descents, for the best elevation
Take compl.t of $\frac{1}{2}$ the angular distance from
the object to the Zenith.

To raise Water
Power P men
Or to $\frac{1}{5}$P Horses
Can raise to 8P ft = F
The No. H hhd in T min.
Or G gall in 60 T sec
OR. H×F = P×8×T min.

Fall of Bodies
Let S = space in ft.
T = time in secon.
V = Vel in ft. p. sec.
Then $\sqrt{S} = 4T = \frac{1}{8}V$
Bodies fall 16 ft in 1st sec.

No. of Shot or Shells in a Pile
Let n = No. in an angular row
m = No. less one in the top row
Then $\overline{n+2} \times \overline{n+1} \times \frac{1}{6}n$ = No. △
And $\overline{n+1} \times \overline{2n+1} \times \frac{1}{6}n$ = No. □
Also $\overline{2n+1+3m} \times \overline{n+1} \times \frac{1}{6}n$ = No. ▭ } Pile

Diam. × 3,1416 = Circum. } Circle
Sq.r Diam. × 0,7854 = Area }
Sq.r Diam. × 3,1416 = Surf. } Sphere
Cub. Diam. × 0,5236 = Solid }

2150,4 cu. In. = 1 Bushel

| English | | French |
|---|---|---|
| Feet 114 | = | 107 Feet |
| 1,0000 | = | 0,9386 |
| 1,0654 | = | 1,0000 |
| lb 108 | = | 100 lb |
| 1,08 | = | 1,00 |
| 1,00 | = | 0,926 |

282 cu. In. = B.G
231. cu. In. = W.G
94 : 77 :: W.G : B.G

| Troy | | Avoirdup. |
|---|---|---|
| 576 lb. | = | 700 lb. |
| 1,000 | = | 0,82274 |
| 1,21545 | = | 1,00000 |
| 82 oz | = | 90 oz |
| 1,00000 | = | 1,09707 |
| 0,91152 | = | 1,00000 |

64 : 9 :: 1
16
64)144(2 1/4
128
16

.75
.75
375
525
.5625
.75
18125
39375
.421875

4 3/4
64 : 9 :: .75
9
64)675(10.
64
35

64 : 144 :: .421875
144
1687500
1687500
421875
593440 00

59.3